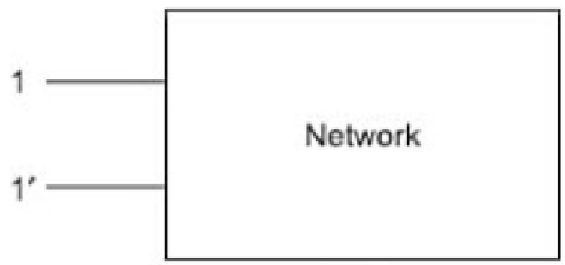
UNIT – III
TWO PORT NETWORK ANALYSIS

Topics: Two port network parameters – Z, Y, ABCD and hybrid parameters and their relations. Concept of transformed network – Two port network parameters using transformed variables- Cascaded networks and analysis of Two-Port networks using the above parameters.

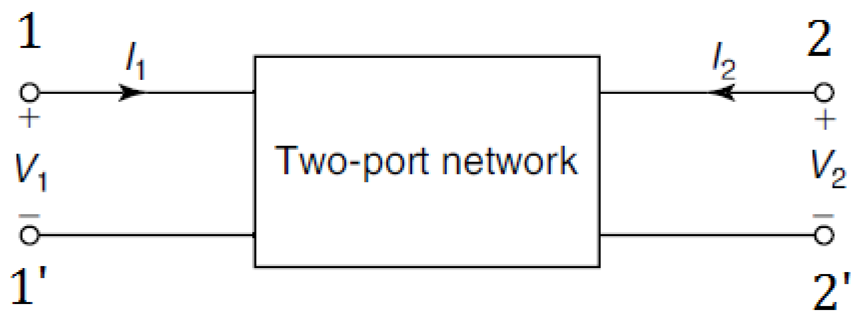
INTRODUCTION:

A pair of terminals at which a signal may enter or leave a network is called a port. A port is defined as any pair of terminals into which energy is supplied or from which energy is withdrawn, or where the network variables may be measured, One such network having only one pair of terminals (1-1') is shown in Fig.(a).



(a)

A **two-port network** is simply a network inside a black box, and the network has only two pairs of accessible terminals; usually one pair represents the input and the other represents the output. Such a building block is very common in electronic systems, communication systems, transmission, and distribution systems. The Figure(b) shows a two-port network, or a two terminal pair network, in which the four terminals have been paired into ports 1-1' and 2-2'.



The terminals 1-1' together constitute a port. Similarly, the terminals 2-2' constitute another port. Two ports containing no sources in their branches are called passive ports; among them are power transmission lines and transformers. Two ports containing sources in their branches are called active ports. A voltage and current assigned to each of the two ports.

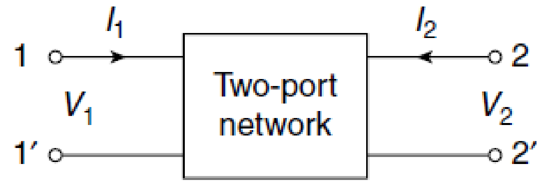
The voltage and current at the input terminals are V_1 and I_1; whereas V_2 and I_2 are specified at the output port. It is also assumed that the currents I_1 and I_2 are entering into the network at the upper terminals 1 and 2, respectively. The variables of the two-port network are V_1, V_2, and I_1, I_2. Two of these are dependent variables; the other two are independent variables. The number of possible combinations generated by the four variables, taken two at a time is six. Thus, there are six possible sets of equations describing a two-port network.

OPEN-CIRCUIT IMPEDANCE PARAMETERS (Z PARAMETERS)

A two-port network is redrawn as in Figure. Here, we consider V_1 and V_2 as dependent variables and I_1 and I_2 as independent variables. Let Z_{11}, Z_{12}, Z_{21} and Z_{22} are the Z-parameters. The voltage V_1 and V_2 in terms of I_1 and I_2 are expressed as follows:

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$



In terms of matrix equations, the following can be obtained:

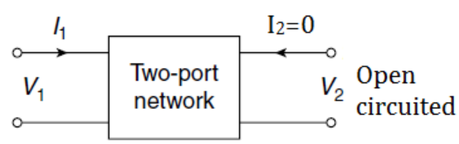
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

where matrix $\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$ is called the impedance matrix.

The individual Z-parameters can be found as indicated in the following. When the output-port is open-circuited, that is, when $I_2 = 0$, the equation will be as follows:

$$V_1 = Z_{11}\ I_1 + 0 \quad \text{that is} \quad Z_{11} = \frac{V_1}{I_1}$$

From equation we will have the following form:

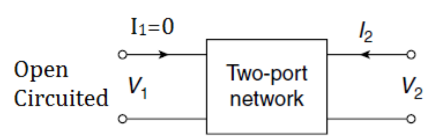


$$V_2 = Z_{21}\ I_1 + 0 \quad \text{that is} \quad Z_{21} = \frac{V_2}{I_1}$$

When the input port is open-circuited, that is, when $I_1 = 0$, then equation (11.1) will be given as follows:

$$V_1 = 0 + Z_{12}\ I_2 \quad \text{that is} \quad Z_{12} = \frac{V_1}{I_2}$$

and from equation we will have the following equation:



$$V_2 = 0 + Z_{22}\ I_2 \quad \text{that is} \quad Z_{22} = \frac{V_2}{I_2}$$

Therefore, the Z-parameters of a two-port network are as follows:

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0}, \quad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0}$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0}, \quad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0}$$

All these parameters are also called open-circuit impedance parameters.

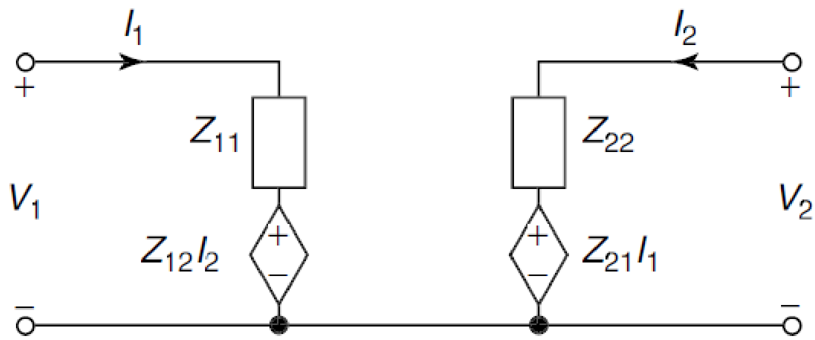
Z_{11} is known as the driving-point impedance at the input port, when output port is open-circuited.

Z_{21} is known as the transfer impedance at the input port, when output port is open-circuited.

Z_{22} is known as the driving-point impedance at the output port, when input port is open-circuited.

Z_{12} is known as the transfer impedance at the output port, when input port is open-circuited.

As these impedance parameters are measured with either the input or output port open-circuited, these are called open-circuit impedance parameters. The equivalent circuit of the two-port network in terms of Z parameters is shown in Fig.

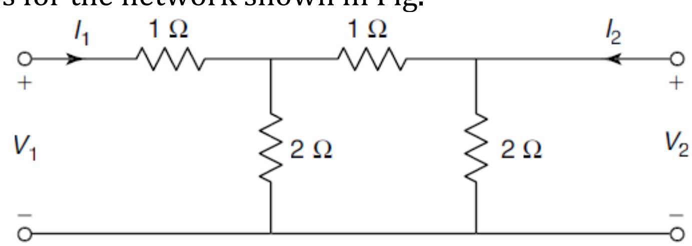


The network is to be reciprocal if $Z_{12} = Z_{21}$

The network is to be symmetrical if $Z_{11} = Z_{22}$

PROBLEMS ON Z-PARAMETERS

1) Find Z-parameters for the network shown in Fig.



SOL:

The network is redrawn as shown in Fig.

Applying KVL to Mesh 1,

$$V_1 = 3I_1 - 2I_3 \qquad \ldots\text{(i)}$$

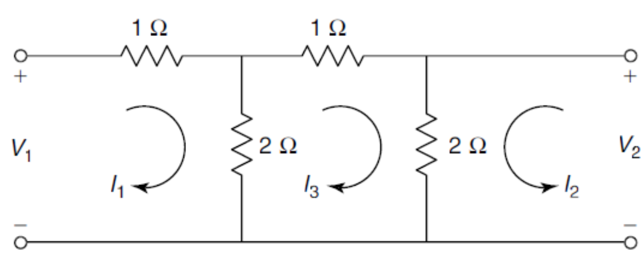
Applying KVL to Mesh 2,

$$V_2 = 2I_2 + 2I_3 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-2I_1 + 2I_2 + 5I_3 = 0$$

$$I_3 = \frac{2}{5}I_1 - \frac{2}{5}I_2 \qquad \ldots\text{(iii)}$$



Substituting Eq. (iii) in Eq. (i),

$$V_1 = 3I_1 - \frac{4}{5}I_1 + \frac{4}{5}I_2$$

$$= \frac{11}{5}I_1 + \frac{4}{5}I_2 \qquad \ldots\text{(iv)}$$

Substituting Eq. (iii) in Eq. (ii),

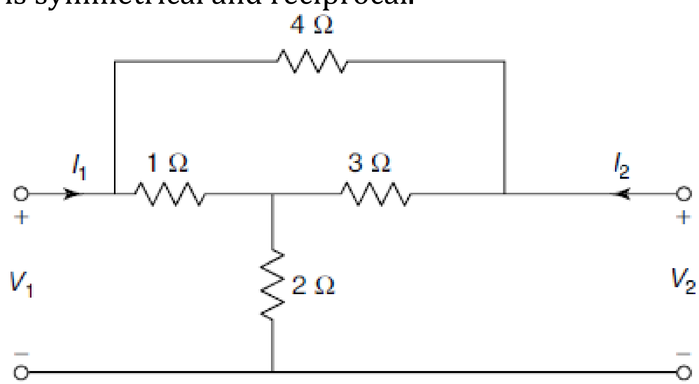
$$V_2 = 2I_2 + \frac{4}{5}I_1 - \frac{4}{5}I_2$$

$$= \frac{4}{5}I_1 + \frac{6}{5}I_2 \qquad \ldots\text{(v)}$$

Comparing Eqs (iv) and (v) with *Z*-parameter equations,

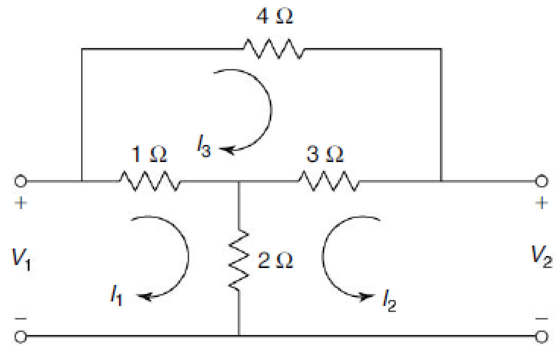
$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{11}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{6}{5} \end{bmatrix}$$

2) Find the open-circuit impedance parameters for the network shown in Fig. Determine whether the network is symmetrical and reciprocal.



SOL:

The network is redrawn as shown in Fig.



Applying KVL to Mesh 1,

$$V_1 - 1(I_1 - I_3) - 2(I_1 + I_2) = 0$$

$$V_1 = 3I_1 + 2I_2 - I_3 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$V_2 - 3(I_2 + I_3) - 2(I_1 + I_2) = 0$$

$$V_2 = 2I_1 + 5I_2 + 3I_3 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-4I_3 - 3(I_2 + I_3) - 1(I_3 - I_1) = 0$$

$$I_1 - 3I_2 + 8I_3 = 0$$

$$I_3 = \frac{1}{8}I_1 - \frac{3}{8}I_2 \qquad \ldots\text{(iii)}$$

Substituting Eq. (iii) in Eq. (i),

$$V_1 = 3I_1 + 2I_2 - \left(\frac{1}{8}I_1 - \frac{3}{8}I_2\right)$$

$$= \frac{23}{8}I_1 + \frac{19}{8}I_2 \qquad \ldots\text{(iv)}$$

Substituting Eq. (iii) in Eq. (ii),

$$V_2 = 2I_1 + 5I_2 + 3\left(\frac{1}{8}I_1 - \frac{3}{8}I_2\right)$$

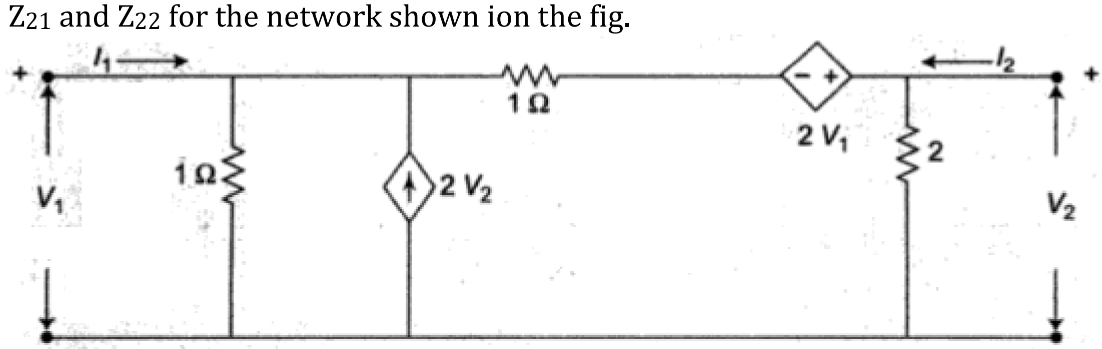
$$= \frac{19}{8}I_1 + \frac{31}{8}I_2 \qquad \ldots\text{(v)}$$

Comparing Eqs (iv) and (v) with Z-parameter equations,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{23}{8} & \frac{19}{8} \\ \frac{19}{8} & \frac{31}{8} \end{bmatrix}$$

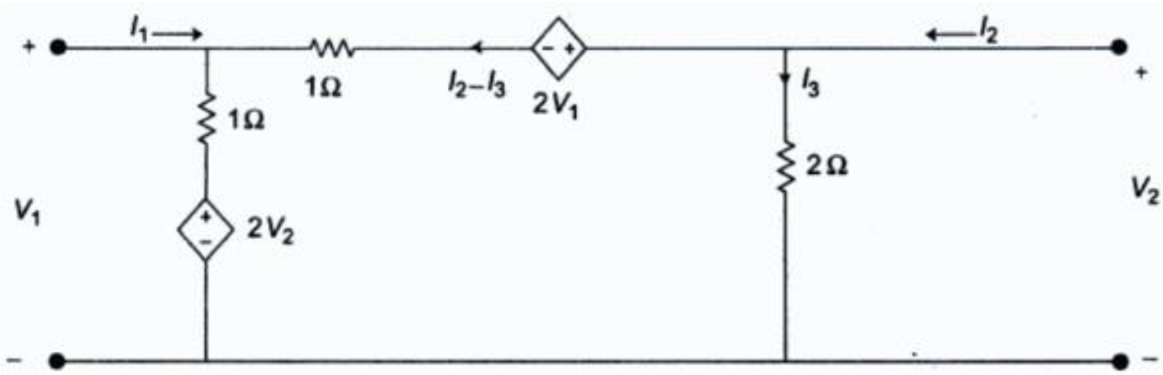
Since $Z_{11} \neq Z_{22}$, the network is not symmetrical.
Since $Z_{12} = Z_{21}$, the network is reciprocal.

3) Find Z_{21} and Z_{22} for the network shown ion the fig.



SOL:

Transforming the dependent current source in to voltage source, the network is shown as



Let I_3 be the current through 2 Ω. KVL to the outer loop.

$$-V_2 + 2V_1 + I_2 - I_3 + V_1 = 0$$

$$-V_2 + 3V_1 + I_2 - I_3 = 0$$

Also $\quad -V_1 + (I_1 + I_2 - I_3) + 2V_2 = 0$

$$V_1 = I_1 + I_2 - I_3 + 2V_2$$

From which

$$-7V_2 - 3I_1 - 2I_2 + 2I_3 = 0$$

where $\quad I_3 = \frac{V_2}{2}$

$$\therefore \quad V_2 = \frac{-I_1}{2} - \frac{I_2}{3}$$

Hence $\quad Z_{21} = \frac{-1}{2}; \; Z_{22} = \frac{-1}{3}$

SHORT-CIRCUIT ADMITTANCE PARAMETERS (or) Y-PARAMETERS

The admittance parameters are also called Y-parameters. To determine Y-parameters, V_1 and V_2 are taken as independent variables and I_1 and I_2 as dependent variables. Port currents I_1 and I_2 are expressed in terms of the voltages V_1 and V_2. The network equations are written as follows:

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{12} V_1 + Y_{22} V_2$$

The equations can be represented in matrix form as follows:

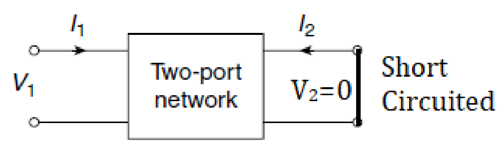
$$[I] = [Y]\,[V]$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

where Y_{11}, Y_{12}, Y_{21} and Y_{22} are admittance parameters and these can be determined as in the following.

When output port is short-circuited, that is, when $V_2 = 0$, equation will give the following form:

$$I_1 = Y_{11}V_1 + 0 \quad \text{or} \quad Y_{11} = \frac{I_1}{V_1}$$

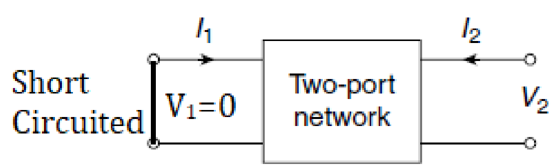


and equation will be calculated as follows:

$$I_2 = Y_{21}V_1 + 0 \quad \text{or} \quad Y_{21} = \frac{I_2}{V_1}$$

When the input port is short-circuited, that is, when $V_1 = 0$, equation can be written as in the following:

$$I_1 = 0 + Y_{12}V_2 \quad \text{or} \quad Y_{12} = \frac{I_1}{V_2}$$



and equation will be given as follows:

$$I_2 = 0 + Y_{22}V_2 \quad \text{or} \quad Y_{22} = \frac{I_2}{V_2}$$

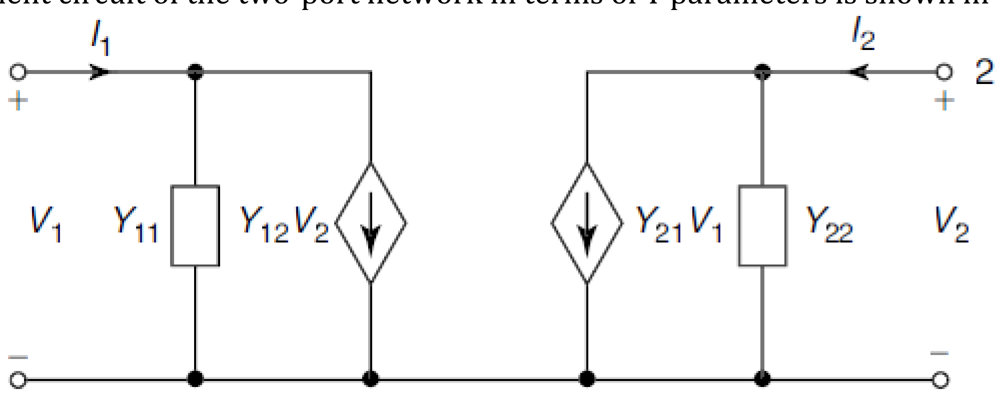
$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2 = 0}$ is also known as the driving-point admittance at input port with output port short-circuited.

$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2 = 0}$ is also known as the transfer admittance at input port with output port short-circuited.

$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = 0}$ is also known as the transfer admittance at output port with input port short-circuited.

$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1 = 0}$ is also known as the driving-point admittance at output port with input port short-circuited.

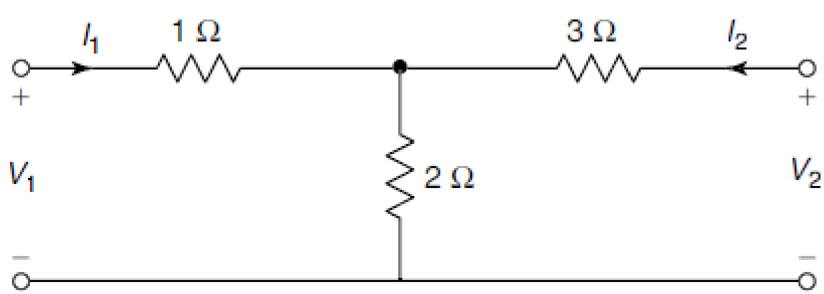
As these admittance parameters are measured with either input or output port short-circuited, these are called short-circuit admittance parameters.
The equivalent circuit of the two-port network in terms of Y parameters is shown in Fig.



If the network is reciprocal if **$Y_{12} = Y_{21}$**
If the network is symmetrical if **$Y_{11} = Y_{22}$**

PROBLEMS ON Y-PARAMETERS

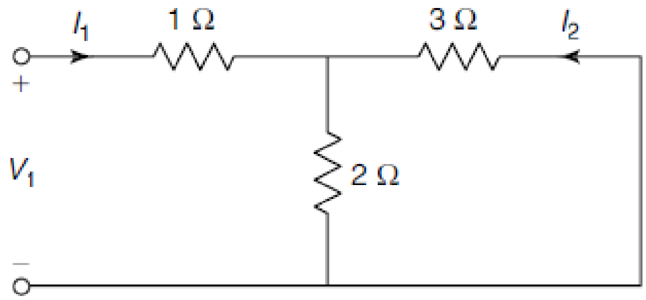
4) Find Y-parameters for the network shown in Fig.



SOL:

Case 1 When the output port is short-circuited, i.e., $V_2 = 0$ as shown in Fig.

$$R_{eq} = 1 + \frac{2\times 3}{2+3} = 1 + \frac{6}{5} = \frac{11}{5}\ \Omega$$



$$V_1 = \frac{11}{5} I_1$$

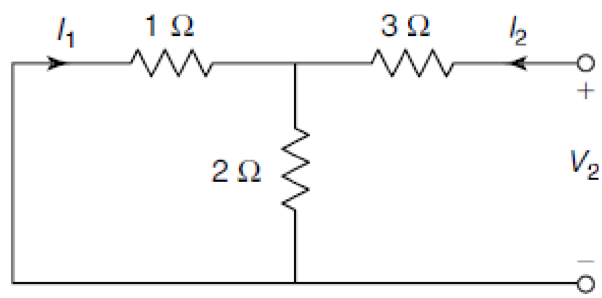
$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{5}{11}\ \mho$$

$$I_2 = \frac{2}{5}(-I_1) = -\frac{2}{5}\times\frac{5}{11}V_1 = -\frac{2}{11}V_1$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\frac{2}{11}\ \mho$$

Case 2 When the input port is short-circuited, i.e., $V_1 = 0$ as shown in Fig.

$$R_{eq} = 3 + \frac{1\times 2}{1+2} = 3 + \frac{2}{3} = \frac{11}{3}\ \Omega$$



$$V_2 = \frac{11}{3} I_2$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{3}{11}\ \mho$$

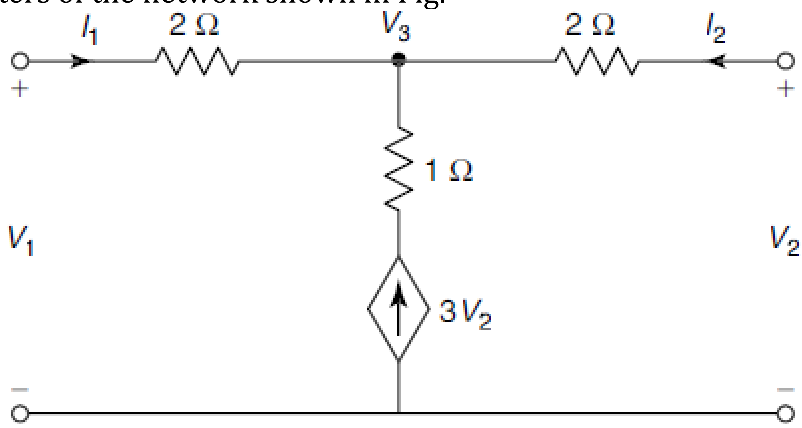
$$I_1 = \frac{2}{3}(-I_2) = -\frac{2}{3}\times\frac{3}{11}V_2 = -\frac{2}{11}V_2$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -\frac{2}{11}\ \mho$$

Hence, the Y-parameters are

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{5}{11} & -\frac{2}{11} \\ -\frac{2}{11} & \frac{3}{11} \end{bmatrix}$$

5) Find Y-parameters of the network shown in Fig.



SOL:

$$I_1 = \frac{V_1 - V_3}{2} = \frac{1}{2}V_1 - \frac{1}{2}V_3 \qquad \ldots(i)$$

$$I_2 = \frac{V_2 - V_3}{2} = \frac{1}{2}V_2 - \frac{1}{2}V_3 \qquad \ldots(ii)$$

Applying KCL at Node 3,

$$I_1 + I_2 + 3V_2 = 0 \qquad \ldots(iii)$$

Substituting Eqs (i) and (ii) in Eq. (iii),

$$\frac{V_1 - V_3}{2} + \frac{V_2 - V_3}{2} + 3V_2 = 0$$

$$2V_3 = V_1 + 7V_2$$

$$V_3 = \frac{1}{2}V_1 + \frac{7}{2}V_2 \qquad \ldots(iv)$$

Substituting Eq. (iv) in Eq. (i),

$$I_1 = \frac{1}{2}V_1 - \frac{1}{2}\left(\frac{1}{2}V_1 + \frac{7}{2}V_2\right)$$

$$= \frac{1}{4}V_1 - \frac{7}{4}V_2 \qquad \ldots(v)$$

Substituting Eq. (iv) in Eq. (ii),

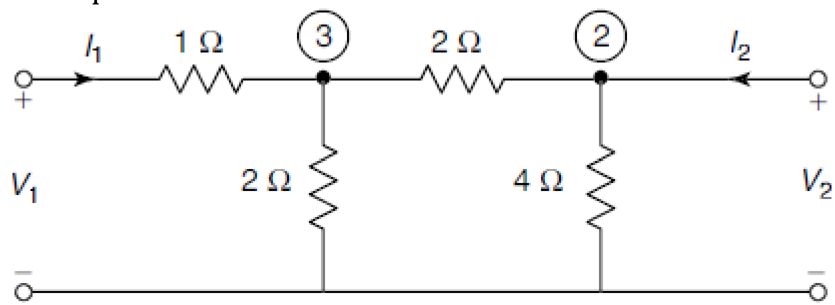
$$I_2 = \frac{1}{2}V_2 - \frac{1}{2}\left(\frac{1}{2}V_1 + \frac{7}{2}V_2\right)$$

$$= -\frac{1}{4}V_1 - \frac{5}{4}V_2 \qquad \ldots(vi)$$

Comparing Eqs (v) and (vi) with Y-parameter equations,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & -\frac{7}{4} \\ -\frac{1}{4} & -\frac{5}{4} \end{bmatrix}$$

6) Determine Y-parameters for the network shown in Fig. Determine whether the network is symmetrical and reciprocal.



SOL:

$$I_1 = \frac{V_1 - V_3}{1} = V_1 - V_3 \qquad \ldots(i)$$

Applying KCL at Node 3,

$$I_1 = \frac{V_3}{2} + \frac{V_3 - V_2}{2} = V_3 - \frac{V_2}{2} \qquad \ldots(ii)$$

Applying KCL at Node 2,

$$I_2 = \frac{V_2}{4} + \frac{V_2 - V_3}{2}$$

$$= \frac{3}{4}V_2 - \frac{V_3}{2} \qquad \ldots(iii)$$

Substituting Eq. (i) in Eq. (ii),

$$V_1 - V_3 = V_3 - \frac{V_2}{2}$$

$$V_3 = \frac{V_1}{2} + \frac{V_2}{4} \qquad \ldots(iv)$$

Substituting Eq. (iv) in Eq. (ii),

$$I_1 = \frac{V_1}{2} + \frac{V_2}{4} - \frac{V_2}{2} = \frac{V_1}{2} - \frac{V_2}{4} \qquad \ldots(v)$$

Substituting Eq. (iv) in Eq. (iii),

$$I_2 = \frac{3}{4}V_2 - \frac{1}{2}\left(\frac{V_1}{2} + \frac{V_2}{4}\right)$$

$$= -\frac{V_1}{4} + \frac{5V_2}{8} \qquad \ldots\text{(vi)}$$

Comparing Eqs (v) and (vi) with Y-parameter equations,

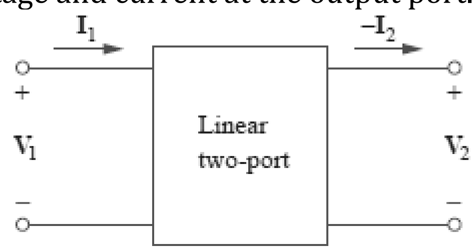
$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{5}{8} \end{bmatrix}$$

Since $Y_{11} \neq Y_{22}$, the network is not symmetrical.
Since $Y_{12} = Y_{21}$, the network is reciprocal.

TRANSMISSION PARAMETERS (or) ABCD PARAMETERS

The transmission parameters or chain parameters or ABCD parameters serve to relate the voltage and current at the input port to voltage and current at the output port.

$$\mathbf{V_1 = AV_2 - BI_2}$$
$$\mathbf{I_1 = CV_2 - DI_2}$$



Here, the negative sign is used with I_2 and not for parameters B and D. The reason the current I_2 carries a negative sign is that in transmission field, the output current is assumed to be coming out of the output port instead of going into the port.
In matrix form, we can write

$$\begin{bmatrix} \mathbf{V_1} \\ \mathbf{I_1} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V_2} \\ -\mathbf{I_2} \end{bmatrix}$$

where matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is called transmission matrix.

The two-port parameters in above equations provide a measure of how a circuit transmits voltage and current from a source to a load. They are useful in the analysis of transmission lines (such as cable and fiber) because they express sending-end variables (V_1 and I_1) in terms of the receiving-end variables (V_2 and $-I_2$). For this reason, they are called transmission parameters. They are also known as ABCD parameters. They are used in the design of telephone systems, microwave networks, and radars.

For a given network, these parameters are determined as follows:

Case 1 When the output port is open-circuited, i.e., $I_2 = 0$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

where A is the reverse voltage gain with the output port open-circuited.

Similarly,
$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

where C is the transfer admittance with the output port open-circuited.

Case 2 When output port is short-circuited, i.e., $V_2 = 0$

$$B = -\left.\frac{V_1}{I_2}\right|_{V_2=0}$$

where B is the transfer impedance with the output port short-circuited.

Similarly,
$$D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$

where D is the reverse current gain with the output port short-circuited.

In simple form,

$$\mathbf{A} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{B} = -\left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0}$$

$$\mathbf{C} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{D} = -\left.\frac{\mathbf{I}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0}$$

Thus, the transmission parameters are called, specifically,

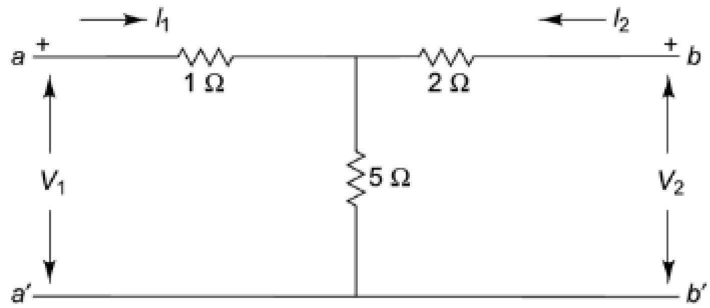
A = Open-circuit voltage ratio
B = Negative short-circuit transfer impedance
C = Open-circuit transfer admittance
D = Negative short-circuit current ratio

A and D are dimensionless, B is in ohms, and C is in siemens. Since the transmission parameters provide a direct relationship between input and output variables, they are very useful in cascaded networks.

The network is to be reciprocal if AD – BC = 1
The network is to be symmetrical if A = D

PROBLEMS ON ABCD-PARAMETERS

7) Find the ABCD parameters for the network shown in the fig.



SOL:

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

When b–b' is open, $I_2 = 0$; $A = \left.\dfrac{V_1}{V_2}\right|_{I_2=0}$

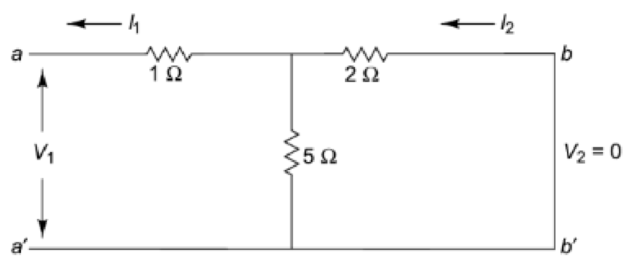
where $V_1 = 6I_1$ and $V_2 = 5I_1$

$$A = \frac{6}{5}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{5}\,\mho$$

When b–b' is short circuited; $V_2 = 0$

$$B = \left.\frac{-V_1}{I_2}\right|_{V_2=0};\ D = \left.\frac{-I_1}{I_2}\right|_{V_2=0}$$



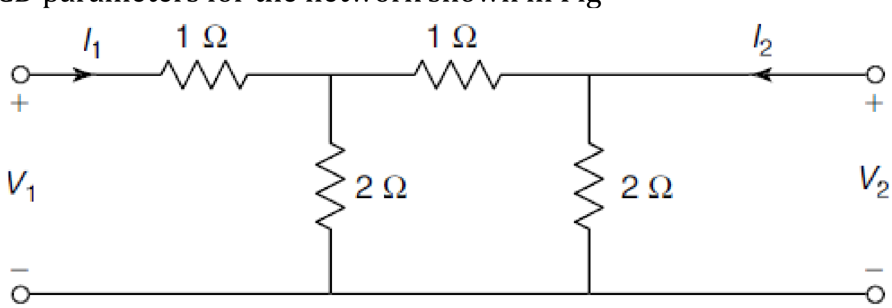
In the circuit, $-I_2 = \dfrac{5}{17}V_1$

$$B = \frac{17}{5}\ \Omega$$

Similarly, $I_1 = \dfrac{7}{17}V_1$ and $-I_2 = \dfrac{5}{17}V_1$

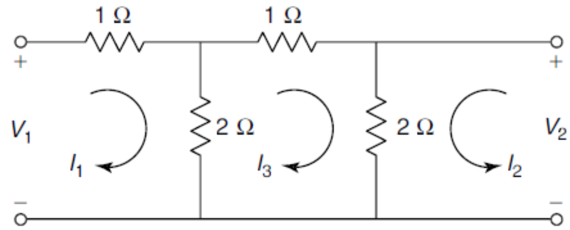
$$D = \frac{7}{5}$$

8) Obtain ABCD parameters for the network shown in Fig



SOL:

The network is redrawn as shown in Fig



Applying KVL to Mesh 1,

$$V_1 = 3I_1 - 2I_3 \quad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$V_2 = 2I_2 + 2I_3 \quad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-2(I_3 - I_1) - I_3 - 2(I_3 + I_2) = 0$$

$$5I_3 = 2I_1 - 2I_2$$

$$I_3 = \frac{2}{5}I_1 - \frac{2}{5}I_2 \quad \ldots\text{(iii)}$$

Substituting Eq. (iii) in Eq. (i),

$$V_1 = 3I_1 - 2\left(\frac{2}{5}I_1 - \frac{2}{5}I_2\right)$$

$$= \frac{11}{5}I_1 + \frac{4}{5}I_2 \quad \ldots\text{(iv)}$$

Substituting Eq. (iii) in Eq. (ii),

$$V_2 = 2I_2 + 2\left(\frac{2}{5}I_1 - \frac{2}{5}I_2\right) = \frac{4}{5}I_1 + \frac{6}{5}I_2$$

$$\frac{4}{5}I_1 = V_2 - \frac{6}{5}I_2$$

$$I_1 = \frac{5}{4}V_2 - \frac{3}{2}I_2 \quad \ldots\text{(v)}$$

Substituting Eq. (v) in Eq. (iv),

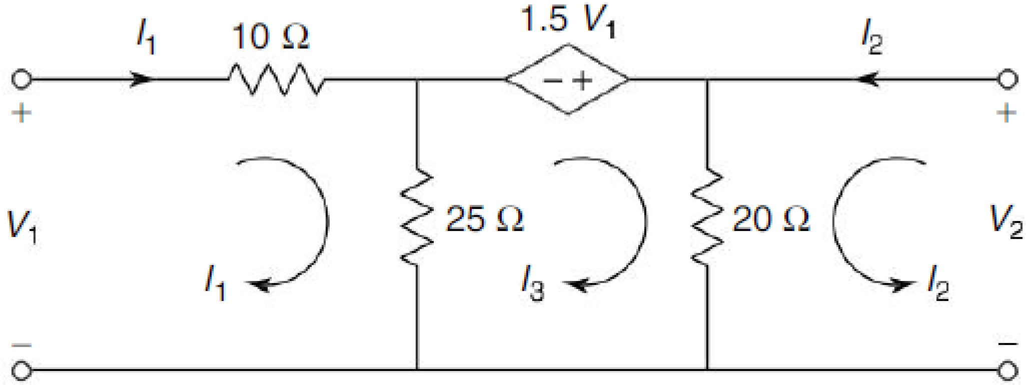
$$V_1 = \frac{11}{5}\left(\frac{5}{4}V_2 - \frac{3}{2}I_2\right) + \frac{4}{5}I_2$$

$$= \frac{11}{4}V_2 - \frac{5}{2}I_2 \quad \ldots\text{(vi)}$$

Comparing Eqs (v) and (vi) with *ABCD* parameter equations,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{11}{4} & \frac{5}{2} \\ \frac{5}{4} & \frac{3}{2} \end{bmatrix}$$

9) Find transmission parameters for the two-port network shown in Fig



SOL:

Applying KVL to Mesh 1,

$$V_1 = 10I_1 + 25(I_1 - I_3)$$
$$= 35I_1 - 25I_3 \quad \ldots(i)$$

Applying KVL to Mesh 2,

$$V_2 = 20(I_2 + I_3)$$
$$= 20I_2 + 20I_3 \quad \ldots(ii)$$

Applying KVL to Mesh 3,

$$-25(I_3 - I_1) + 1.5V_1 - 20(I_2 + I_3) = 0$$
$$-25I_3 + 25I_1 + 1.5(35I_1 - 25I_3) - 20I_2 - 20I_3 = 0$$
$$-25I_3 + 25I_1 + 52.5I_1 - 37.5I_3 - 20I_2 - 20I_3 = 0$$
$$82.5I_3 = 77.5I_1 - 20I_2$$
$$I_3 = 0.94I_1 - 0.24I_2 \quad \ldots(iii)$$

Substituting Eq. (iii) in Eq. (i),

$$V_1 = 35I_1 - 25(0.94I_1 - 0.24I_2)$$
$$= 11.5I_1 + 6I_2 \quad \ldots(iv)$$

Substituting Eq. (iii) in Eq. (ii),

$$V_2 = 20I_2 + 20(0.94I_1 - 0.24I_2)$$
$$= 18.8I_1 + 15.2I_2 \quad \ldots(v)$$

From Eq. (v),

$$I_1 = 0.053V_2 - 0.81I_2 \quad \ldots(vi)$$

Substituting Eq. (vi) in Eq. (iv),

$$V_1 = 11.5(0.053V_2 - 0.81I_2) + 6I_2$$
$$= 0.61V_2 - 3.32I_2 \quad \ldots(vii)$$

Comparing Eqs (vi) and (vii) with *ABCD* parameter equations,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.61 & -3.32 \\ 0.053 & -0.81 \end{bmatrix}$$

INVERSE TRANSMISSION PARAMETERS (or) A'B'C'D' PARAMETERS

The inverse transmission parameters serve to relate the voltage and current at the output port to the voltage and current at the input port.

$$V_2 = A' V_1 - B' I_1$$

$$I_2 = C' V_1 - D' I_1$$

In matrix form, we can write

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

where matrix $\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}$ is called the *inverse transmission matrix*.

For a given network, these parameters are determined as follows:

Case 1 When the input port is open-circuited, i.e., $I_1 = 0$

$$A' = \left. \frac{V_2}{V_1} \right|_{I_1=0}$$

where A' is the forward voltage gain with the input port open-circuited.

Similarly,

$$C' = \left. \frac{I_2}{V_1} \right|_{I_1=0}$$

where C' is the transfer admittance with the input port open-circuited.

Case 2 When the input port is short-circuited, i.e., $V_1 = 0$

$$B' = -\left. \frac{V_2}{I_1} \right|_{V_1=0}$$

where B' is the transfer impedance with the input port short-circuited.

Similarly,

$$D' = -\left. \frac{I_2}{I_1} \right|_{V_1=0}$$

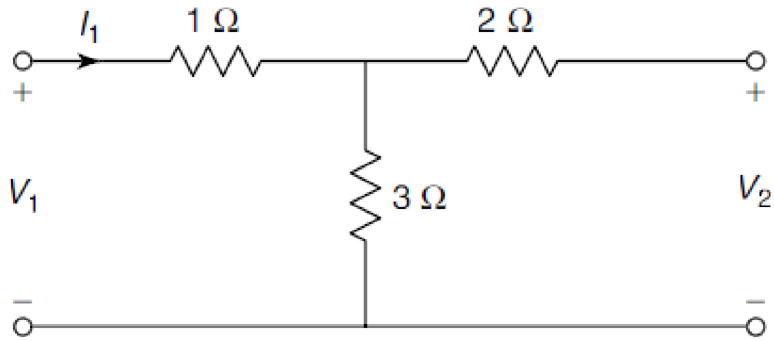
where D' is the forward current gain with the input port short-circuited.

The network is to be reciprocal if A'D' – B'C' = 1

The network is to be symmetrical if A' = D'

PROBLEMS ON A'B'C'D'-PARAMETERS

10) Find the inverse transmission parameters for the network shown in Fig



SOL:

Applying KVL to Mesh 1,

$$V_1 = 4I_1 + 3I_2 \quad \text{...(i)}$$

Applying KVL to Mesh 2,

$$V_2 = 3I_1 + 5I_2 \quad \text{...(ii)}$$

Hence, $$3I_2 = V_1 - 4I_1$$

$$I_2 = \frac{1}{3}V_1 - \frac{4}{3}I_1 \quad \text{...(iii)}$$

Substituting Eq. (iii) in Eq. (ii),

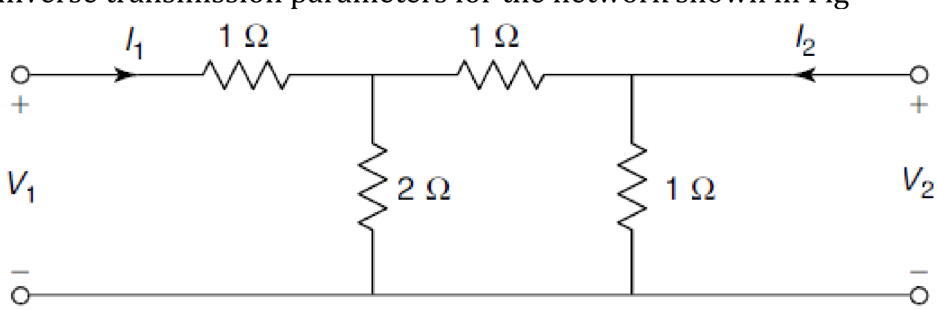
$$V_2 = 3I_1 + 5\left(\frac{1}{3}V_1 - \frac{4}{3}I_1\right)$$

$$= \frac{5}{3}V_1 - \frac{11}{3}I_1 \quad \text{...(iv)}$$

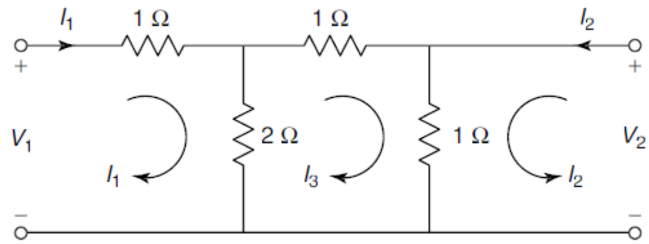
Comparing Eqs (iii) and (iv) with inverse transmission parameter equations,

$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} \frac{5}{3} & \frac{11}{3} \\ \frac{1}{3} & \frac{4}{3} \end{bmatrix}$$

11) Find the inverse transmission parameters for the network shown in Fig



The network is redrawn as shown in Fig.



Applying KVL to Mesh 1,

$$V_1 = 3I_1 - 2I_3 \quad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$V_2 = I_2 + I_3 \quad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-2I_1 + I_2 + 4I_3 = 0$$

$$I_3 = \frac{1}{2}I_1 - \frac{1}{4}I_2 \quad \ldots\text{(iii)}$$

Substituting Eq. (iii) in Eq. (i),

$$V_1 = 3I_1 - 2\left(\frac{1}{2}I_1 - \frac{1}{4}I_2\right)$$

$$= 2I_1 + \frac{1}{2}I_2 \quad \ldots\text{(iv)}$$

Substituting Eq. (iii) in Eq. (ii),

$$V_2 = I_2 + \frac{1}{2}I_1 - \frac{1}{4}I_2$$

$$= \frac{1}{2}I_1 + \frac{3}{4}I_2 \quad \ldots\text{(v)}$$

Rewriting Eq. (iv),

$$\frac{1}{2}I_2 = V_1 - 2I_1$$

$$I_2 = 2V_1 - 4I_1 \quad \ldots\text{(vi)}$$

Substituting the Eq. (vi) in Eq. (v),

$$V_2 = \frac{1}{2}I_1 + \frac{3}{4}(2V_1 - 4I_1)$$

$$= \frac{3}{2}V_1 - \frac{5}{2}I_1 \quad \ldots\text{(vii)}$$

Comparing Eqs (vi) and (vii) with inverse transmission parameter equations,

$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & \frac{5}{2} \\ 2 & 4 \end{bmatrix}$$

HYBRID PARAMETERS (or) h - PARAMETERS

The hybrid parameters of a two-port network may be defined by expressing the voltage of input port V_1 and current of output port I_2 in terms of current of input port I_1 and voltage of output port V_2.

$$\mathbf{V_1 = h_{11}I_1 + h_{12}V_2}$$
$$\mathbf{I_2 = h_{21}I_1 + h_{22}V_2}$$

the matrix form is given as in the following:

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

The individual h parameters can be defined by setting $I_1 = 0$ and $V_2 = 0$.

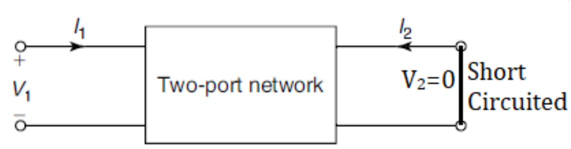
Case 1 When the output port is short-circuited i.e., $V_2 = 0$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

where h_{11} is the short-circuit input impedance.

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

where h_{21} is the short-circuit forward current gain.



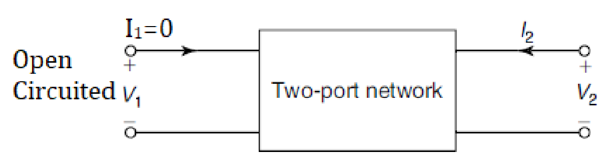
Case 2 When the input port is open-circuited, i.e., $I_1 = 0$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

where h_{12} is the open-circuit reverse voltage gain.

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

where h_{22} is the open-circuit output admittance.



In simple form,

$$\mathbf{h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}}, \qquad \mathbf{h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}}$$

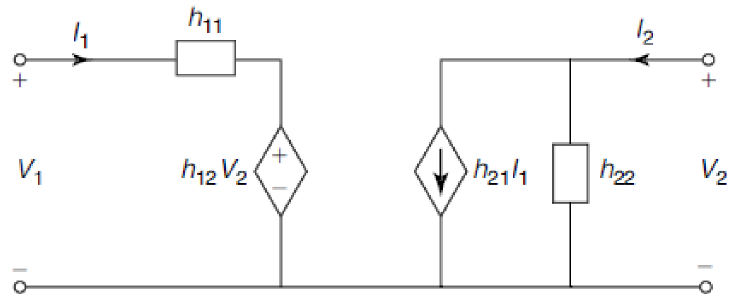
$$\mathbf{h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}}, \qquad \mathbf{h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}}$$

The network is to be reciprocal if $h_{12} = -h_{21}$

The network is to be symmetrical if $h_{11} h_{22} - h_{12} h_{21} = 1$

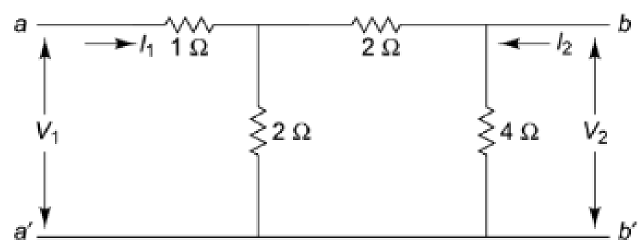
It is evident from Eq. that the parameters represent impedance, a voltage gain, a current gain, and admittance, respectively. Therefore, they are called the hybrid parameters.

The equivalent circuit of a two-port network in terms of hybrid parameters is shown in Fig



PROBLEMS ON h-PARAMETERS

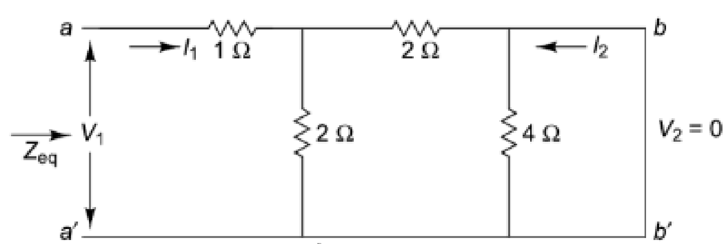
12) Find the h parameters of the network shown in the fig.



SOL:

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} ; h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} ; h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} ; h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

If port b–b' is short circuited, $V_2 = 0$. The circuit is shown in Fig.



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} ; V_1 = I_1 Z_{eq}$$

Z_{eq} the equivalent impedance as viewed from the port a–a' is 2 Ω

$$V_1 = I_1 2 \text{ V}$$

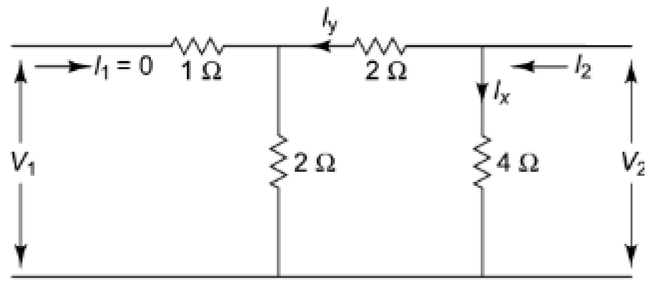
$$h_{11} = \frac{V_1}{I_1} = 2\ \Omega$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \quad \text{when } V_2 = 0; -I_2 = \frac{I_1}{2}$$

$$h_{21} = -\frac{1}{2}$$

If port a–a' is let open, $I_1 = 0$. The circuit is shown in Fig. Then

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$



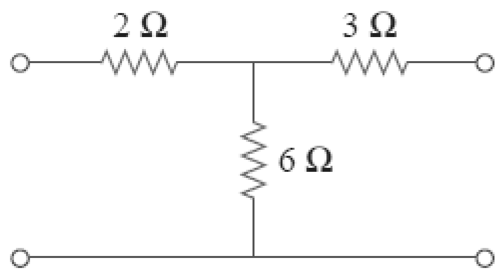
$$V_1 = I_Y 2;\; I_Y = \frac{I_2}{2}$$

$$V_2 = I_X 4;\; I_X = \frac{I_2}{2}$$

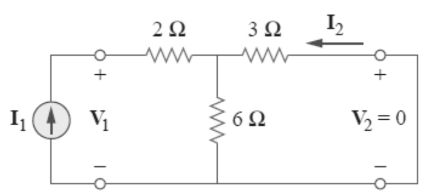
$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{1}{2}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{2}\,\mho$$

13) Find the hybrid parameters for the two-port network of Fig.



SOL:



(a)

To find $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$, we short-circuit the output port and connect a current source $\mathbf{I}_1$ to the input port as shown in Fig. (a). From Fig.

$$\mathbf{V}_1 = \mathbf{I}_1(2 + 3 \parallel 6) = 4\mathbf{I}_1$$

Hence,

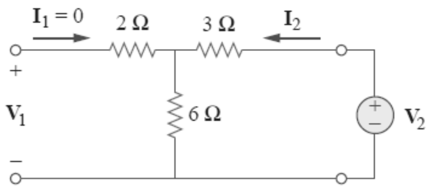
$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 4\ \Omega$$

Also, from Fig. (a) we obtain, by current division,

$$-\mathbf{I}_2 = \frac{6}{6+3}\mathbf{I}_1 = \frac{2}{3}\mathbf{I}_1$$

Hence,

$$\mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{2}{3}$$



(b)

To obtain $\mathbf{h}_{12}$ and $\mathbf{h}_{22}$, we open-circuit the input port and connect a voltage source $\mathbf{V}_2$ to the output port as in Fig.(b). By voltage division,

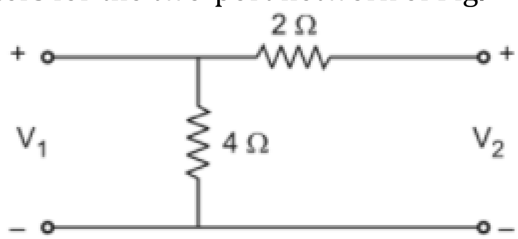
$$\mathbf{V}_1 = \frac{6}{6+3}\mathbf{V}_2 = \frac{2}{3}\mathbf{V}_2$$

Hence, $$\mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{2}{3}$$

Also, $$\mathbf{V}_2 = (3+6)\mathbf{I}_2 = 9\mathbf{I}_2$$

$$\mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{9}\text{ S}$$

14) Find the hybrid parameters for the two-port network of Fig.



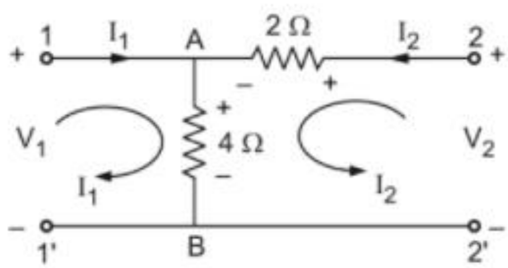
SOL:

By definition, h-parameters are given by,

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

Using loop analysis technique, the network can be drawn as shown in the Fig. (a).



Applying KVL to loop 1-A-B-1'-1, we get,

$$-4I_1 - 4I_2 + V_1 = 0$$

$$\therefore \quad V_1 = 4I_1 + 4I_2$$

Applying KVL to loop 2-A-B-2′-2, we get,

$$-2I_2 - 4I_2 - 4I_1 + V_2 = 0$$

$$\therefore \quad V_2 = 4I_1 + 6I_2$$

Rearranging terms in equation

$$6I_2 = V_2 - 4I_1$$

$$\therefore \quad I_2 = \frac{1}{6}V_2 - \frac{4}{6}I_1$$

$$\text{i.e.} \quad I_2 = \left(-\frac{2}{3}\right)I_1 + \left(\frac{1}{6}\right)V_2$$

Substituting value of I_2 in equation we get,

$$V_1 = 4I_1 + 4\left[\left(\frac{1}{6}\right)V_2 + \left(-\frac{2}{3}\right)I_1\right]$$

$$V_1 = 4I_1 + \frac{4}{6}V_2 - \frac{8}{3}I_1$$

$$V_1 = \left(\frac{4}{3}\right)I_1 + \left(\frac{2}{3}\right)V_2$$

Hence

$$[h] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{4}{3} & \frac{2}{3} \\ -\frac{2}{3} & \frac{1}{6} \end{bmatrix}$$

INVERSE HYBRID PARAMETERS (or) g – PARAMETERS

The inverse hybrid parameters of a two-port network may be defined by expressing the current of the input port I_1 and voltage of the output port V_2 in terms of the voltage of the input port V_1 and the current of the output port I_2.

$$I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

In matrix form, we can write

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

The individual g parameters can be defined by setting $V_1 = 0$ and $I_2 = 0$.

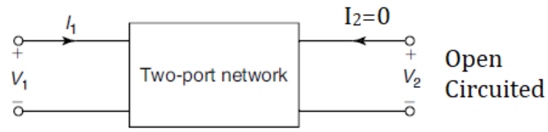
Case 1 When the output port is open-circuited, i.e., $I_2 = 0$

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$$

where g_{11} is the open-circuit input admittance.

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$$

where g_{21} is the open-circuit forward voltage gain.



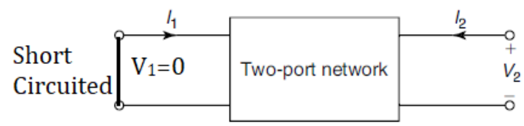
Case 2 When the input port is short-circuited, i.e., $V_1 = 0$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$

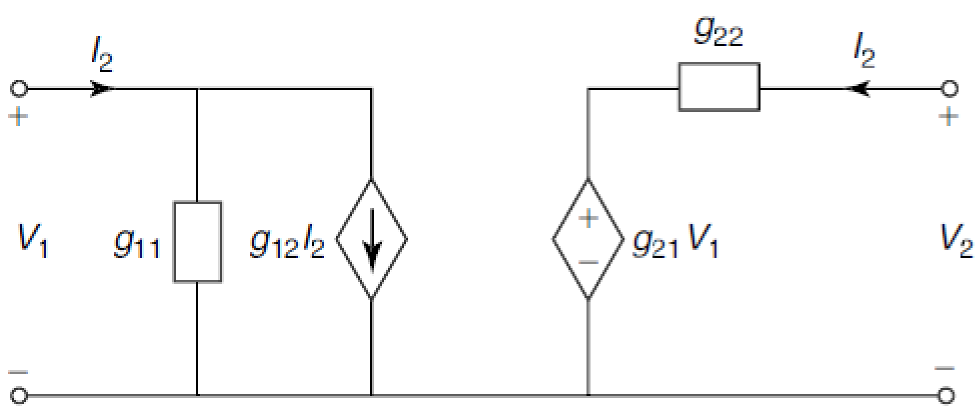
where g_{12} is the short-circuit reverse current gain.

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

where g_{22} is the short-circuit output impedance.



The equivalent circuit of a two-port network in terms of inverse hybrid parameters is shown in Fig.



The network is to be reciprocal if $g_{12} = -g_{21}$

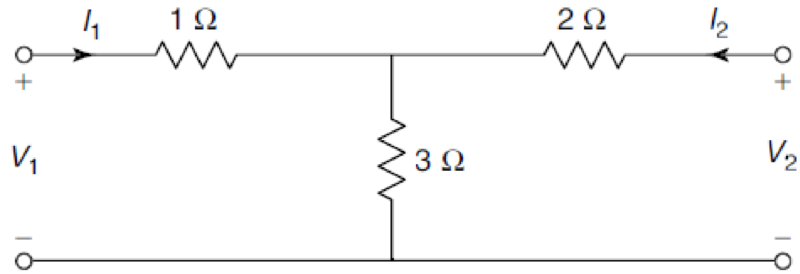
The network is to be symmetrical if $g_{11}g_{22} - g_{12}g_{21} = 1$

NOTE: Conditions for reciprocity and symmetry

Parameter	Condition for Reciprocity	Condition for Symmetry
Z	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$
T	$AD - BC = 1$	$A = D$
T'	$A'D' - B'C' = 1$	$A' = D'$
h	$h_{12} = -h_{21}$	$h_{11}h_{22} - h_{12}h_{21} = 1$
g	$g_{12} = -g_{21}$	$g_{11}g_{22} - g_{12}g_{21} = 1$

PROBLEMS ON g-PARAMETERS

15) Find g-parameters for the network shown in Fig.



SOL:

Applying KVL to Mesh 1,

$$V_1 = 4\,I_1 + 3\,I_2 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$V_2 = 3\,I_1 + 5\,I_2 \qquad \ldots\text{(ii)}$$

Hence,

$$4\,I_1 = V_1 - 3\,I_2$$

$$I_1 = \frac{1}{4}V_1 - \frac{3}{4}I_2 \qquad \ldots\text{(iii)}$$

Substituting Eq. (iii) in Eq. (ii),

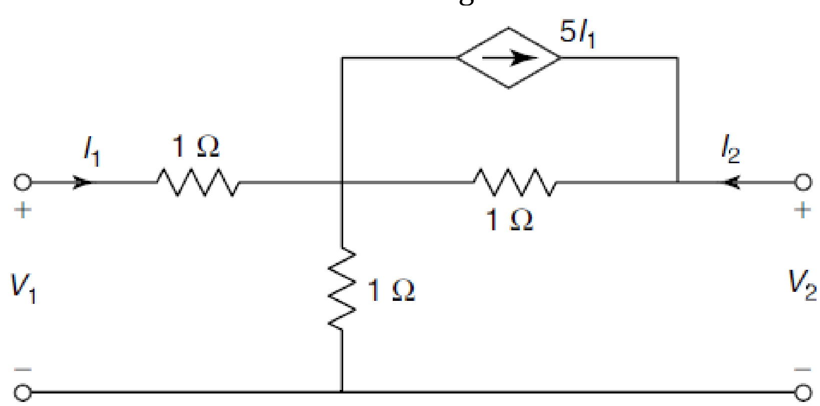
$$V_2 = 3\left(\frac{1}{4}V_1 - \frac{3}{4}I_2\right) + 5\,I_2$$

$$= \frac{3}{4}V_1 + \frac{11}{4}I_2 \qquad \ldots\text{(iv)}$$

Comparing Eqs (iii) and (iv) with *g*-parameter equations,

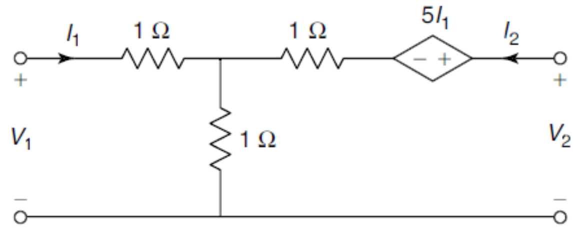
$$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & -\frac{3}{4} \\ \frac{3}{4} & \frac{11}{4} \end{bmatrix}$$

16) Find g-parameters for the network shown in Fig.



SOL:

The network is redrawn using source transformation as shown in Fig



Applying KVL to Mesh 1,

$$V_1 - 1\,I_1 - 1\,(I_1 + I_2) = 0$$

$$V_1 = 2\,I_1 + I_2 \qquad \ldots(\text{i})$$

Applying KVL to Mesh 2,

$$V_2 - 5\,I_1 - 1\,I_2 - 1\,(I_2 + I_1) = 0$$

$$V_2 = 6\,I_1 + 2\,I_2 \quad \ldots(\text{ii})$$

Hence, $2\,I_1 = V_1 - I_2$

$$I_1 = \frac{1}{2}V_1 - \frac{1}{2}I_2 \qquad \ldots(\text{iii})$$

Substituting Eq (iii) in Eq. (ii),

$$V_2 = 6\left(\frac{1}{2}V_1 - \frac{1}{2}I_2\right) + 2\,I_2$$

$$= 3\,V_1 - I_2 \qquad \ldots(\text{iv})$$

Comparing Eqs (iii) and (iv) the *g*- parameter equations,

$$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 3 & -1 \end{bmatrix}$$

SUMMARY OF TWO PORT NETWORK PARAMETERS:

Name of the parameters	Port variables		Equations	Condition of reciprocity	Condition of symmetry
	Express (Dependent)	Interms of (Independent)			
Open circuit impedance i.e. z-parameters	V_1 , V_2	I_1 , I_2	$V_1 = z_{11} I_1 + z_{12} I_2$ $V_2 = z_{21} I_1 + z_{22} I_2$	$z_{12} = z_{21}$	$z_{11} = z_{22}$
Short circuit admittance i.e. y-parameters	I_1 , I_2	V_1 , V_2	$I_1 = y_{11} V_1 + y_{12} V_2$ $I_2 = y_{21} V_1 + y_{22} V_2$	$y_{12} = y_{21}$	$y_{11} = y_{22}$
h-parameters	V_1 , I_2	I_1 , V_2	$V_1 = h_{11} I_1 + h_{12} V_2$ $I_2 = h_{21} I_1 + h_{22} V_2$	$h_{12} = -h_{21}$	$h_{11}h_{22} - h_{12}h_{21} = 1$
ABCD or transmission parameters	V_1 , I_1	V_2 , $-I_2$	$V_1 = AV_2 + B(-I_2)$ $I_1 = CV_2 + D(-I_2)$	$AD - BC = 1$	$A = D$
Inverse hybrid or g-parameters	I_1 , V_2	V_1 , I_2	$I_1 = g_{11} V_1 + g_{12} I_2$ $V_2 = g_{21} V_1 + g_{22} I_2$	$g_{12} = -g_{21}$	$g_{11}g_{22} - g_{12}g_{21} = 1$
Inverse transmission parameters	V_2 , I_2	V_1 , $-I_1$	$V_2 = A' V_1 + B'(-I_1)$ $I_2 = C' V_1 + D'(-I_2)$	$A'D' - B'C' = 1$	$A' = D'$

INTER-RELATIONSHIPS BETWEEN THE TWO-PORT NETWORK PARAMETERS (OR) CORRELATION OF TWO-PORT NETWORK PARAMETERSL

CASE-I : Z-PARAMETERS IN TERMS OF OTHER PARAMETERS

i) Z-parameters in Terms of Y-parameters :

We know that, the equations of Y-parameters are

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{----- (i)}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{----- (ii)}$$

From equation(ii), $Y_{22} V_2 = I_2 - Y_{21} V_1$

$$V_2 = \frac{I_2}{Y_{22}} - \frac{Y_{21}}{Y_{22}} V_1$$

Substitute V_2 in equation(i)

$$I_1 = Y_{11}V_1 + Y_{12}\left(\frac{I_2}{Y_{22}} - \frac{Y_{21}}{Y_{22}}V_1\right)$$

$$Y_{22}I_1 = Y_{11}Y_{22}V_1 + Y_{12}I_2 - Y_{12}Y_{21}V_1$$

$$Y_{22}I_1 = (Y_{11}Y_{22} - Y_{12}Y_{21})V_1 + Y_{12}I_2$$

Let $\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$

$$Y_{22}I_1 = \Delta Y\, V_1 + Y_{12}I_2$$

$$\Delta Y\, V_1 = Y_{22}I_1 - Y_{12}I_2$$

$$V_1 = \frac{Y_{22}}{\Delta Y} I_1 - \frac{Y_{12}}{\Delta Y} I_2 \quad --- \text{(iii)}$$

From equation(i), $Y_{11} V_1 = I_1 - Y_{12} V_2$

$$V_1 = \frac{I_1}{Y_{11}} - \frac{Y_{12}}{Y_{11}} V_2$$

Substitute V_1 in equation (ii)

$$I_2 = Y_{21}\left(\frac{I_1}{Y_{11}} - \frac{Y_{12}}{Y_{11}} V_2\right) + Y_{22}V_2$$

$$Y_{11}I_2 = Y_{21}I_1 + V_2(Y_{11}Y_{22} - Y_{12}Y_{21})$$

$$Y_{11}I_2 = Y_{21}I_1 + V_2\, \Delta Y$$

$$V_2 = -\frac{Y_{21}}{\Delta Y} I_1 + \frac{Y_{11}}{\Delta Y} I_2 \quad \text{---- (iv)}$$

We know that, the equations of Z-parameters are

$$V_1 = Z_{11}\ I_1 + Z_{12}\ I_2 \quad \text{------- (v)}$$

$$V_2 = Z_{21}\ I_1 + Z_{22}\ I_2 \quad \text{------- (vi)}$$

Comparing eq(v), (vi) with eq(iii), eq(iv)

$$Z_{11} = \frac{Y_{22}}{\Delta Y} \qquad Z_{12} = -\frac{Y_{12}}{\Delta Y}$$

$$Z_{21} = -\frac{Y_{21}}{\Delta Y} \qquad Z_{22} = \frac{Y_{11}}{\Delta Y}$$

ii) Z-parameters in Terms of ABCD-parameters :

We know that

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Rewriting the second equation,

$$V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2$$

Comparing with $V_2 = Z_{21} I_1 + Z_{22} I_2,$

$$Z_{21} = \frac{1}{C}$$

$$Z_{22} = \frac{D}{C}$$

Also, $V_1 = A\left[\frac{1}{C} I_1 + \frac{D}{C} I_2\right] - BI_2 = \frac{A}{C} I_1 + \left[\frac{AD}{C} - B\right] I_2 = \frac{A}{C} I_1 + \left[\frac{AD - BC}{C}\right] I_2$

Comparing with $V_1 = Z_{11} I_1 + Z_{12} I_2,$

$$Z_{11} = \frac{A}{C}$$

Hence

$$Z_{12} = \frac{AD - BC}{C}$$

$$Z_{11} = \frac{A}{C} \qquad Z_{12} = \frac{AD - BC}{C}$$

$$Z_{21} = \frac{1}{C} \qquad Z_{22} = \frac{D}{C}$$

iii) Z-parameters in Terms of Hybrid (h)-parameters :

We know that
$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

Rewriting the second equation,
$$V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2$$

Comparing with
$$V_2 = Z_{21} I_1 + Z_{22} I_2,$$
$$Z_{21} = -\frac{h_{21}}{h_{22}}$$
$$Z_{22} = \frac{1}{h_{22}}$$

Also, $V_1 = h_{11} I_1 + h_{12}\left[-\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2\right] = h_{11} I_1 + \frac{h_{12}}{h_{22}} I_2 - \frac{h_{12}h_{21}}{h_{22}} I_1 = \left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}\right] I_1 + \frac{h_{12}}{h_{22}} I_2$

Comparing with
$$V_1 = Z_{11} I_1 + Z_{12} I_2,$$
$$Z_{11} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} = \frac{\Delta h}{h_{22}}$$
$$Z_{12} = \frac{h_{12}}{h_{22}}$$

Hence

$$Z_{11} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} = \frac{\Delta h}{h_{22}} \qquad Z_{12} = \frac{h_{12}}{h_{22}}$$

$$Z_{21} = -\frac{h_{21}}{h_{22}} \qquad Z_{22} = \frac{1}{h_{22}}$$

CASE-II : Y-PARAMETERS IN TERMS OF OTHER PARAMETERS

iv) Y-parameters in Terms of Z-parameters :

We know that, the equations of Z-parameters are

$$V_1 = Z_{11}\ I_1 + Z_{12}\ I_2 \qquad \text{-------}\ (i)$$

$$V_2 = Z_{21}\ I_1 + Z_{22}\ I_2 \qquad \text{-------}\ (ii)$$

From equation(ii)

$$I_2 = \frac{1}{Z_{22}} V_2 - \frac{Z_{21}}{Z_{22}} I_1$$

Substitute I_2 in equation(i)

$$V_1 = Z_{11}\, I_1 + \ Z_{12} \left(\frac{1}{Z_{22}} V_2 - \frac{Z_{21}}{Z_{22}} I_1\right)$$

$$Z_{22}\ V_1 = (Z_{11}Z_{22} - Z_{12}Z_{21}) I_1 + \ Z_{12}\, V_2$$

Let $\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$

$$Z_{22}\ V_1 = \Delta Z\, I_1 + \ Z_{12}\, V_2$$

$$I_1 = \frac{Z_{22}}{\Delta Z}\ V_1 - \ \frac{Z_{12}}{\Delta Z}\ V_2 \qquad ---(iii)$$

From equation(i)

$$I_1 = \frac{1}{Z_{11}} V_1 - \frac{Z_{12}}{Z_{11}} I_2$$

Substitute I_1 in equation(ii)

$$V_2 = Z_{21} \left(\frac{1}{Z_{11}} V_1 - \frac{Z_{12}}{Z_{11}} I_2\right) + \ Z_{22}\, I_2$$

$$Z_{11} V_2 = Z_{21} V_1 + \ (Z_{11}Z_{22} - Z_{12}Z_{21}) I_2$$

$$Z_{11} V_2 = Z_{21} V_1 + \ \Delta Z\ I_2$$

$$I_2 = -\ \frac{Z_{21}}{\Delta Z}\ V_1 + \ \frac{Z_{11}}{\Delta Z}\ V_2 \qquad ---(iv)$$

We know that, the equations of Y-parameters are

$$I_1 = Y_{11}\ V_1 + Y_{12}\ V_2 \qquad \text{-----}\ (v)$$

$$I_2 = Y_{21}\ V_1 + Y_{22}\ V_2 \qquad \text{-----}\ (vi)$$

Comparing eq(v), (vi) with eq(iii), eq(iv)

$$Y_{11} = \frac{Z_{22}}{\Delta Z} \qquad Y_{12} = -\frac{Z_{12}}{\Delta Z}$$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z} \qquad Y_{22} = \frac{Z_{11}}{\Delta Z}$$

v) Y-parameters in Terms of ABCD-parameters :

We know that $V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

Rewriting the first equation,

$$I_2 = -\frac{1}{B}V_1 + \frac{A}{B}V_2$$

Comparing with $I_2 = Y_{21}V_1 + Y_{22}V_2$,

$$Y_{21} = -\frac{1}{B} \qquad Y_{22} = \frac{A}{B}$$

$$I_1 = CV_2 - D\left[-\frac{1}{B}V_1 + \frac{A}{B}V_2\right] = \frac{D}{B}V_1 + \left[\frac{BC - AD}{B}\right]V_2$$

Comparing with $I_1 = Y_{11}V_1 + Y_{12}V_2$,

$$Y_{11} = \frac{D}{B}$$

$$Y_{12} = \frac{BC - AD}{B} = -\frac{AD - BC}{B}$$

vi) Y-parameters in Terms of Hybrid (h)-parameters :

We know that $V_1 = h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$

Rewriting the first equation,

$$I_1 = \frac{1}{h_{11}}V_1 - \frac{h_{12}}{h_{11}}V_2$$

Comparing with $I_1 = Y_{11}V_1 + Y_{12}V_2$,

$$Y_{11} = \frac{1}{h_{11}} \qquad Y_{12} = -\frac{h_{12}}{h_{11}}$$

Also $$I_2 = h_{21}\left[\frac{1}{h_{11}}V_1 - \frac{h_{12}}{h_{11}}V_2\right] + h_{22}V_2 = \frac{h_{21}}{h_{11}}V_1 + \left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}}\right]V_2$$

Comparing with $I_2 = Y_{21}V_1 + Y_{22}V_2$,

$$Y_{21} = \frac{h_{21}}{h_{11}}$$

$$Y_{22} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}} = \frac{\Delta h}{h_{11}}$$

CASE-III : ABCD-PARAMETERS IN TERMS OF OTHER PARAMETERS

vii) ABCD-parameters in Terms of Z-parameters :

We know that $$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Rewriting the second equation,

$$I_1 = \frac{1}{Z_{21}}V_2 - \frac{Z_{22}}{Z_{21}}I_2$$

Comparing with $I_1 = CV_2 - DI_2$,

$$C = \frac{1}{Z_{21}}$$

$$D = \frac{Z_{22}}{Z_{21}}$$

Also, $$V_1 = Z_{11}\left[\frac{1}{Z_{21}}V_2 - \frac{Z_{22}}{Z_{21}}I_2\right] + Z_{12}I_2 = \frac{Z_{11}}{Z_{21}}V_2 - \frac{Z_{22}Z_{11}}{Z_{21}}I_2 + Z_{12}I_2$$

$$= \frac{Z_{11}}{Z_{21}}V_2 - \left[\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}\right]I_2$$

Comparing with $V_1 = AV_2 - BI_2$,

$$A = \frac{Z_{11}}{Z_{21}}$$

$$B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}} = \frac{\Delta Z}{Z_{21}}$$

viii) ABCD-parameters in Terms of Y-parameters :

We know that $I_1 = Y_{11} V_1 + Y_{12} V_2$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

Rewriting the second equation,

$$V_1 = -\frac{Y_{22}}{Y_{21}} V_2 + \frac{1}{Y_{21}} I_2$$

Comparing with $V_1 = AV_2 - BI_2$,

$$A = -\frac{Y_{22}}{Y_{21}}$$

$$B = -\frac{1}{Y_{21}}$$

Also, $I_1 = Y_{11}\left[-\frac{Y_{22}}{Y_{21}} V_2 + \frac{1}{Y_{21}} I_2\right] + Y_{12} V_2 = \left[\frac{Y_{12} Y_{21} - Y_{11} Y_{22}}{Y_{21}}\right] V_2 + \frac{Y_{11}}{Y_{21}} I_2$

Comparing with $I_1 = CV_2 - DI_2$,

$$C = \frac{Y_{12} Y_{21} - Y_{11} Y_{22}}{Y_{21}} = -\frac{\Delta Y}{Y_{21}}$$

$$D = -\frac{Y_{11}}{Y_{21}}$$

ix) ABCD-parameters in Terms of Hybrid (h)-parameters :

We know that $V_1 = h_{11} I_1 + h_{12} V_2$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

Rewriting the second equation,

$$I_1 = -\frac{h_{22}}{h_{21}} V_2 + \frac{1}{h_{21}} I_2$$

Comparing with $I_1 = CV_2 - DI_2$,

$$C = -\frac{h_{22}}{h_{21}}$$

$$D = -\frac{1}{h_{21}}$$

Also, $V_1 = h_{11}\left[\frac{1}{h_{21}} I_2 - \frac{h_{22}}{h_{21}} V_2\right] + h_{12} V_2 = \left[\frac{h_{12} h_{21} - h_{11} h_{22}}{h_{21}}\right] V_2 + \frac{h_{11}}{h_{21}} I_2$

Comparing with $V_1 = AV_2 - BI_2$,

$$A = \frac{h_{12}\,h_{21} - h_{11}\,h_{22}}{h_{21}} = -\frac{\Delta h}{h_{21}}$$

$$B = -\frac{h_{11}}{h_{21}}$$

<u>CASE-IV : HYBRID (h)-PARAMETERS IN TERMS OF OTHER PARAMETERS</u>

x) Hybrid (h)-parameters in Terms of Z-parameters :

We know that $V_1 = Z_{11}\,I_1 + Z_{12}\,I_2$

$$V_2 = Z_{21}\,I_1 + Z_{22}\,I_2$$

Rewriting the second equation,

$$I_2 = -\frac{Z_{21}}{Z_{22}}I_1 + \frac{1}{Z_{22}}V_2$$

Comparing with $I_2 = h_{21}\,I_1 + h_{22}\,V_2$,

$$h_{21} = -\frac{Z_{21}}{Z_{22}}$$

$$h_{22} = \frac{1}{Z_{22}}$$

Also, $V_1 = Z_{11}\,I_1 + Z_{12}\left[-\frac{Z_{21}}{Z_{22}}I_1 + \frac{1}{Z_{22}}V_2\right] = \left[\frac{Z_{11}\,Z_{22} - Z_{12}\,Z_{21}}{Z_{22}}\right]I_1 + \frac{Z_{12}}{Z_{22}}V_2$

Comparing with $V_1 = h_{11}\,I_1 + h_{12}\,V_2$,

$$h_{11} = \frac{Z_{11}\,Z_{22} - Z_{12}\,Z_{21}}{Z_{22}} = \frac{\Delta Z}{Z_{22}}$$

$$h_{12} = \frac{Z_{12}}{Z_{22}}$$

xi) Hybrid (h)-parameters in Terms of Y-parameters :

We know that $I_1 = Y_{11}\,V_1 + Y_{12}\,V_2$

$$I_2 = Y_{21}\,V_1 + Y_{22}\,V_2$$

Rewriting the first equation,

$$V_1 = \frac{1}{Y_{11}}I_1 - \frac{Y_{12}}{Y_{11}}V_2$$

Comparing with $V_1 = h_{11} I_1 + h_{12} V_2$,

$$h_{11} = \frac{1}{Y_{11}}$$

$$h_{12} = -\frac{Y_{12}}{Y_{11}}$$

Also, $$I_2 = Y_{21}\left[\frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2\right] + Y_{22} V_2 = \left[\frac{Y_{11} Y_{22} - Y_{12} Y_{21}}{Y_{11}}\right] V_2 + \frac{Y_{21}}{Y_{11}} I_1$$

Comparing with $I_2 = h_{21} I_1 + h_{22} V_2$,

$$h_{21} = \frac{Y_{22}}{Y_{11}}$$

$$h_{22} = \frac{Y_{11} Y_{22} - Y_{12} Y_{21}}{Y_{11}} = \frac{\Delta Y}{Y_{11}}$$

xii) Hybrid (h)-parameters in Terms of ABCD-parameters :

We know that $V_1 = AV_2 - BI_2$

$$I_1 = CV_2 - DI_2$$

Rewriting the second equation,

$$I_2 = -\frac{1}{D} I_1 + \frac{C}{D} V_2$$

Comparing with $I_2 = h_{21} I_1 + h_{22} V_2$,

$$h_{21} = -\frac{1}{D}$$

$$h_{22} = \frac{C}{D}$$

Also, $$V_1 = AV_2 - B\left[-\frac{1}{D} I_1 + \frac{C}{D} V_2\right] = \frac{B}{D} I_1 + \left[\frac{AD - BC}{D}\right] V_2$$

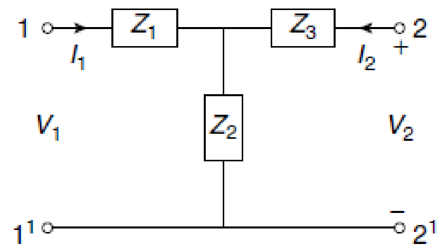
Comparing with $V_1 = h_{11} I_1 + h_{12} V_2$,

$$h_{11} = \frac{B}{D}$$

$$h_{12} = \frac{AD - BC}{D} = \frac{\Delta T}{D}$$

T-CIRCUIT REPRESENTATION OF TWO-PORT NETWORK

A *T*-connected two-port network is shown in Figure



Applying KVL, we get the following form:

$$V_1 = I_1 (Z_1 + Z_2) + I_2 Z_2$$

$$V_2 = I_1 Z_2 + I_2 (Z_2 + Z_3)$$

Comparing equations with the general equations of *Z*-parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

and $$V_2 = Z_{21} I_1 + Z_{22} I_2$$

We get the following,

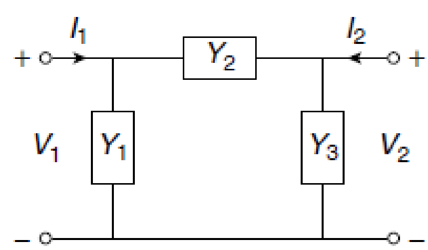
$$Z_{11} = Z_1 + Z_2$$
$$Z_{12} = Z_2 = Z_{21}$$
$$Z_{22} = Z_2 + Z_3$$

From the above equations, the T-network impedances are

$$Z_2 = Z_{12} = Z_{21}$$
$$Z_1 = Z_{11} - Z_{12}$$
$$Z_3 = Z_{22} - Z_{12}$$

Π-CIRCUIT REPRESENTATION OF TWO-PORT NETWORK

A two-port network can also be represented by an equivalent π circuit as shown in Figure



Applying KCL, the nodal equation can be written as in the following:

$$I_1 = V_1(Y_1 + Y_2) - V_2 Y_2$$
$$I_2 = -V_1 Y_2 + V_2 (Y_2 + Y_3)$$

Comparing the equations with the general equations of *Y*-parameters, we get the following forms:

$$Y_{11} = Y_1 + Y_2$$
$$Y_{12} = Y_1 = -Y_2$$
$$Y_{22} = Y_2 + Y_3$$

Therefore,

$$Y_2 = -Y_{12} = -Y_{21}$$
$$Y_1 = Y_{11} + Y_{12}$$
$$Y_3 = Y_{22} + Y_{12}$$

17) The Z parameters of a two-port network are Z_{11} = 20 Ω, Z_{22} = 30 Ω, Z_{12} = Z_{21} = 10 Ω. Find Y and ABCD parameters.

SOL:

$$\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21} = (20)(30) - (10)(10) = 500$$

***Y*-parameters**

$$Y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{30}{500} = \frac{3}{50}\,℧, \qquad Y_{12} = -\frac{Z_{12}}{\Delta Z} = -\frac{10}{500} = -\frac{1}{50}\,℧$$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z} = -\frac{10}{500} = -\frac{1}{50}\,℧, \qquad Y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{20}{500} = \frac{2}{50}\,℧$$

Hence, the *Y*-parameters are

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{3}{50} & -\frac{1}{50} \\ -\frac{1}{50} & \frac{2}{50} \end{bmatrix}$$

***ABCD* parameters**

$$A = \frac{Z_{11}}{Z_{21}} = \frac{20}{10} = 2, \qquad B = \frac{\Delta Z}{Z_{21}} = \frac{500}{10} = 50$$

$$C = \frac{1}{Z_{21}} = \frac{1}{10} = 0.1, \qquad D = \frac{Z_{22}}{Z_{21}} = \frac{30}{10} = 3$$

Hence, the *ABCD* parameters are

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 2 & 50 \\ 0.1 & 3 \end{bmatrix}$$

18) The Currents I_1 and I_2 entering at Port 1 and Port 2 respectively of a two-port network are given by the following equations:

$$I_1 = 0.5V_1 - 0.2V_2$$

$$I_2 = -0.2V_1 + V_2$$

Find Y, Z and ABCD parameters for the network.

SOL:

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = 0.5\,℧, \qquad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -0.2\,℧$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -0.2\,℧, \qquad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = 1\,℧$$

Hence, the Y-parameters are

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 0.5 & -0.2 \\ -0.2 & 1 \end{bmatrix}$$

***Z*-parameters**

$$\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21} = (0.5)(1) - (-0.2)(-0.2) = 0.46$$

$$Z_{11} = \frac{Y_{22}}{\Delta Y} = \frac{1}{0.46} = 2.174\ \Omega, \qquad Z_{12} = -\frac{Y_{12}}{\Delta Y} = -\frac{(-0.2)}{0.46} = 0.434\ \Omega$$

$$Z_{21} = -\frac{Y_{21}}{\Delta Y} = -\frac{(-0.2)}{0.46} = 0.434\ \Omega, \qquad Z_{22} = \frac{Y_{11}}{\Delta Y} = \frac{0.5}{0.46} = 1.087\ \Omega$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 2.174 & 0.434 \\ 0.434 & 1.087 \end{bmatrix}$$

***ABCD* parameters**

$$A = -\frac{Y_{22}}{Y_{21}} = -\frac{1}{-0.2} = 5, \qquad B = -\frac{1}{Y_{21}} = -\frac{1}{-0.2} = 5$$

$$C = -\frac{\Delta Y}{Y_{21}} = -\frac{0.46}{-0.2} = 2.3, \qquad D = -\frac{Y_{11}}{Y_{21}} = -\frac{0.5}{-0.2} = 2.5$$

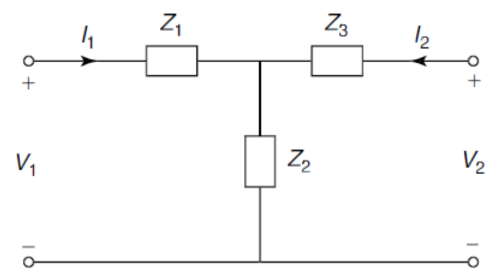
Hence, the $ABCD$ parameters are

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ 2.3 & 2.5 \end{bmatrix}$$

19) The Z-parameters of a two-port network are: Z_{11} = 10 Ω, Z_{12} = Z_{21} = 5 Ω, Z_{22} = 20 Ω. Find the equivalent T-network.

SOL:

The T-network is shown in Fig



Applying KVL to Mesh 1,

$$V_1 = (Z_1 + Z_2)I_1 + Z_2 I_2 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$V_2 = Z_2 I_1 + (Z_2 + Z_3)I_2 \qquad \ldots\text{(ii)}$$

Comparing Eqs (i) and (ii) with Z parameter equations,

$$Z_{11} = Z_1 + Z_2 = 10$$

$$Z_{12} = Z_2 = 5$$

$$Z_{21} = Z_2 = 5$$

$$Z_{22} = Z_2 + Z_3 = 20$$

Solving the above equations,

$$Z_1 = 5\ \Omega$$

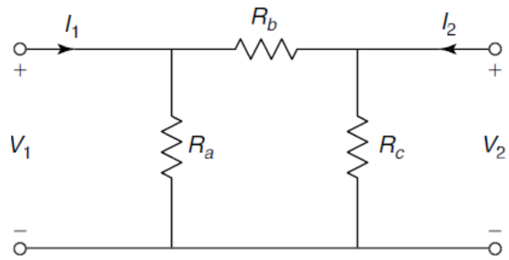
$$Z_2 = 5\ \Omega$$

$$Z_3 = 15\ \Omega$$

20) The Admittance parameters of a two-port network are $Y_{11} = 0.09$ mho, $Y_{12} = Y_{21} = -0.05$ mho and $Y_{22} = 0.07$ mho. Find the equivalent Π-network.

SOL:

The pi network is shown in Fig



Applying KCL at Node, 1,

$$I_1 = \frac{V_1}{R_a} + \frac{V_1 - V_2}{R_b}$$

$$= \left(\frac{1}{R_a} + \frac{1}{R_b}\right)V_1 - \frac{1}{R_b}V_2 \qquad \ldots\text{(i)}$$

Applying KCL at Node 2,

$$I_2 = \frac{V_2}{R_c} + \frac{V_2 - V_1}{R_b}$$

$$= -\frac{1}{R_b}V_1 + \left(\frac{1}{R_B} + \frac{1}{R_c}\right)V_2 \qquad \ldots\text{(ii)}$$

Comparing Eqs (i) and (ii) with Y-parameter equations,

$$Y_{11} = \frac{1}{R_a} + \frac{1}{R_b} = 0.09$$

$$Y_{12} = -\frac{1}{R_b} = -0.05$$

$$Y_{21} = -\frac{1}{R_b} = -0.05$$

$$Y_{22} = \frac{1}{R_b} + \frac{1}{R_c} = 0.07$$

Solving the above equations,

$$R_a = 25\ \Omega$$

$$R_b = 20\ \Omega$$

$$R_c = 50\ \Omega$$

CONCEPT OF TRANSFORMED NETWORK

In any network, the parameters can be solved by using differential equation method and using Laplace transform method. To analyze any network on a transform basis, the only additional step required is to represent all the network elements in terms of' complex impedance and admittance with associated initial energy sources. Therefore, we define the transform impedance and admittance and then find out their expression for each of the circuit elements like resistance, inductance, and capacitance.

For a single element, the transform impedance is defined as the ratio of the transform of the element voltage to the transform of the element current for zero initial current in an inductor and zero initial voltage in a capacitor.

$$Z(s) = \frac{V(s)}{I(s)}$$

Similarly, the transform admittance is defined as the ratio of transform of the element current for zero initial current in an inductor and zero initial voltage in a capacitor.

$$Y(s) = \frac{I(s)}{V(s)} = \frac{1}{Z(s)}$$

i) Resistance: For a resistance, the voltage and current are related in the time domain by Ohm's law.

$$V = Ri \quad \text{or} \quad i = GV \quad \text{where } G = \frac{1}{R}$$

The corresponding transform equations are

$$V(s) = RI(s)$$
$$I(s) = GV(s)$$

The transform impedance of the resistor

$$Z(s) = \frac{V(s)}{I(s)} = R$$

Similarly, the transform admittance of' the resistor

$$Y(s) = \frac{I(s)}{V(s)} = G$$

From the above results, we can say that the resistor is frequency independent to the complex frequency.

ii) Inductance: For inductance, the time domain relation between the current and voltage inductance as

$$v(t) = L\frac{di(t)}{dt}$$
$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(t)\,d(t).$$

The equivalent transform equation for the voltage

$$V(s) = L[SI(s) - i(0+)]$$

$$LSI(s) = V(s) + Li(0+)$$

where i(0+) is the initial current present in the inductor at i = 0+.
If the initial current i(0+) = 0, the transform impedance for the inductor

$$Z(s) = \frac{V(s)}{I(s)} = SL$$

and the transform admittance becomes

$$Y(s) = \frac{I(s)}{V(s)} = \frac{1}{LS}$$

iii) Capacitance: For capacitance, the time domain relation between voltage and current is expressed as

$$i(t) = c\frac{dv(t)}{dt}$$

$$v(t) = \frac{1}{c}\int_{-\infty}^{t} i_c(t)dt$$

The equivalent transform equation for the voltage expression is

$$V(s) = \frac{1}{c}\left[\frac{I(s)}{s} + \frac{q(0+)}{s}\right]$$

The above equation becomes

$$\frac{1}{cs}I(s) = V(s) - \frac{v_c(0+)}{S}$$

By considering the initial charge on the capacitor zero.
The equation becomes

$$\frac{1}{cs}I(s) = V(s)$$

The transform impedance of the capacitor is the ratio of transform voltage V(s) to the transform current I(s) and is

$$Z(s) = \frac{V(s)}{I(s)} = \frac{1}{cs}$$

The transform admittance of the capacitor is the ratio of transform current I(s) to the transform voltage V(s) is

$$Y(s) = \frac{I(s)}{V(s)} = cs$$

TWO-PORT PARAMETERS USING TRANSFORMED VARIABLES

For the two-port network without internal sources, the driving point impedance function at port 1-1' is the ratio of the transform voltage at port 1-1' to the transform current at the same port.

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)}$$

Similarly, the driving point impedance at port 2-2' is the ratio of transform current at the same port

$$Z_{22} = \frac{V_2(s)}{I_2(s)}$$

For the two-port network, the driving point admittance is defined as the ratio of the transform current at any port to the transform voltage at the same port.

$$Y_{11}(s) = \frac{I_1(s)}{V_1(s)}$$

$$Y_{22}(s) = \frac{I_2(s)}{V_2(s)}$$

The four other network functions are called transfer functions. These functions give the relation between voltage or current at one port to the voltage or current at the other port as shown hereunder.

i) Voltage Transfer Ratio: It is defined as the ratio of voltage transform at one port to the voltage transform at the other port and is denoted by G(s)

$$G_{21}(s) = \frac{V_2(s)}{V_1(s)}$$

$$G_{12}(s) = \frac{V_1(s)}{V_2(s)}$$

ii) Current Transfer Ratio: It is defined as the ratio of current transform at one port to the current transform at the other port and is denoted by $\boldsymbol{\alpha}$(s).

$$\alpha_{12}(s) = \frac{I_1(s)}{I_2(s)}$$

$$\alpha_{21}(s) = \frac{I_2(s)}{I_1(s)}$$

iii) Transfer Impedance: It is defined as the ratio of voltage transform at one port to the current transform at the other port. and is denoted by Z(s).

$$Z_{21} = \frac{V_2(s)}{I_1(s)}$$

$$Z_{12}(s) = \frac{V_1(s)}{I_2(s)}$$

iv) Transfer Admittance: It is defined as the ratio of current transform at one port to the voltage transform at the other port and is denoted by Y(s).

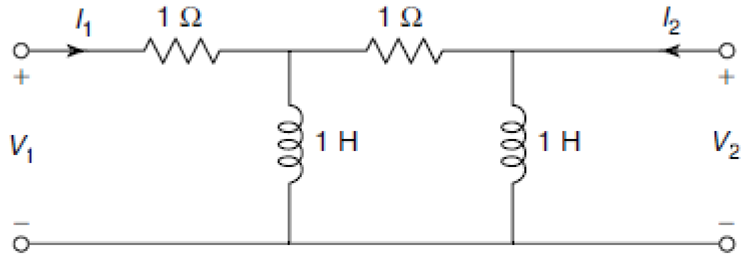
$$Y_{21}(s) = \frac{I_2(s)}{V_1(s)}$$

$$Y_{12}(s) = \frac{I_1(s)}{V_2(s)}$$

The above network functions are found by forming the system of equations using nodal or mesh analysis and taking the transforms of equations by setting the initial condition to zero and solving for ratio of the response to excitation.

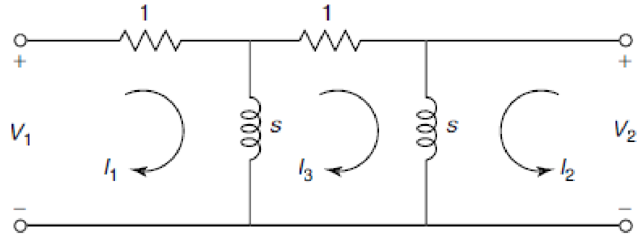
PROBLEMS ON TRANSFORMED NETWORKS

21) Find the Z-parameters for the network shown in Fig.



SOL:

The transformed network is shown in Fig.



Applying KVL to Mesh 1,

$$V_1 = (s+1)I_1 - sI_3 \qquad \ldots\text{(i)}$$

Applying KVL to Mesh 2,

$$V_2 = sI_2 + sI_3 \qquad \ldots\text{(ii)}$$

Applying KVL to Mesh 3,

$$-sI_1 + sI_2 + (2s+1)I_3 = 0$$

$$I_3 = \frac{s}{2s+1}I_1 - \frac{s}{2s+1}I_2 \quad \ldots \text{(iii)}$$

Substituting Eq. (iii) in Eq. (i),

$$V_1 = (s+1)I_1 - s\left(\frac{s}{2s+1}I_1 - \frac{s}{2s+1}I_2\right)$$

$$= \left(\frac{s^2+3s+1}{2s+1}\right)I_1 + \left(\frac{s^2}{2s+1}\right)I_2 \qquad \ldots\text{(iv)}$$

Substituting Eq. (iii) in Eq. (ii),

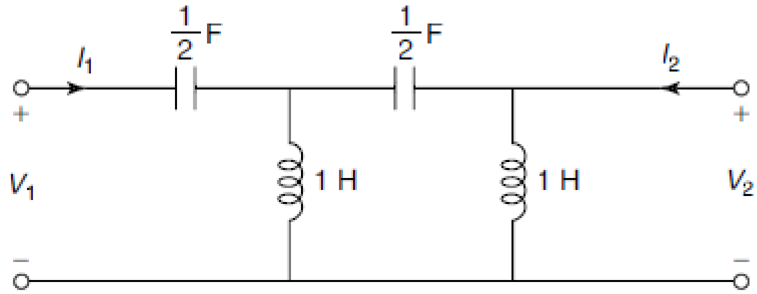
$$V_2 = sI_2 + s\left(\frac{s}{2s+1}I_1 - \frac{s}{2s+1}I_2\right)$$

$$= \left(\frac{s^2}{2s+1}\right)I_1 + \left(\frac{s^2+s}{2s+1}\right)I_2 \qquad \ldots\text{(v)}$$

Comparing Eqs (iv) and (v) with Z-parameter equations,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{s^2+3s+1}{2s+1} & \frac{s^2}{2s+1} \\ \frac{s^2}{2s+1} & \frac{s^2+s}{2s+1} \end{bmatrix}$$

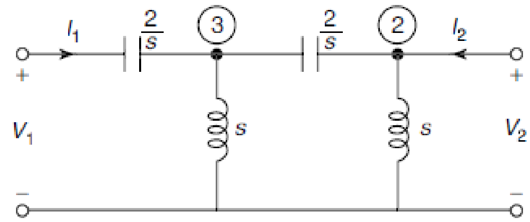
22) Determine Y-parameters for the network shown in Fig.



SOL:

The transformed network as shown in Fig.

From Fig.



$$I_1 = \frac{V_1 - V_3}{\frac{2}{s}}$$

$$= \frac{s}{2}V_1 - \frac{s}{2}V_3 \qquad \ldots\text{(i)}$$

Applying KCL at Node 3,

$$\frac{s}{2}(V_1 - V_3) = \frac{V_3}{s} + \frac{s}{2}(V_3 - V_2)$$

$$\frac{s}{2}V_3 + \frac{1}{s}V_3 + \frac{s}{2}V_3 = \frac{s}{2}V_1 + \frac{s}{2}V_2$$

$$V_3 = \frac{s^2}{2(s^2+1)}V_1 + \frac{s^2}{2(s^2+1)}V_2 \qquad \ldots\text{(ii)}$$

Substituting Eq. (ii) in Eq. (i),

$$I_1 = \frac{s}{2}V_1 - \frac{s}{2}\left[\frac{s^2}{2(s^2+1)}V_1 + \frac{s^2}{2(s^2+1)}V_2\right]$$

$$= \left[\frac{s}{2} - \frac{s^3}{4(s^2+1)}\right]V_1 - \frac{s^3}{4(s^2+1)}V_2$$

$$= \frac{s^3+2s}{4(s^2+1)}V_1 - \frac{s^3}{4(s^2+1)}V_2 \qquad \ldots\text{(iii)}$$

Applying KCL at Node 2,

$$I_2 = \frac{V_2}{s} + \frac{s}{2}(V_2 - V_3)$$

$$= \frac{s^2+2}{2s}V_2 - \frac{s}{2}V_3 \qquad \ldots\text{(iv)}$$

Substituting Eq. (ii) in Eq. (iv),

$$I_2 = \frac{s^2+2}{2s}V_2 - \frac{s}{2}\left[\frac{s^2}{2(s^2+1)}V_1 + \frac{s^2}{2(s^2+1)}V_2\right]$$

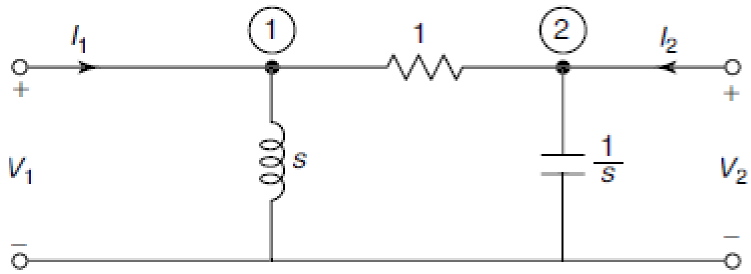
$$= -\frac{s^3}{4(s^2+1)}V_1 + \left[\frac{s^2+2}{2s} - \frac{s^3}{4(s^2+1)}\right]V_2$$

$$= -\frac{s^3}{4(s^2+1)}V_1 + \frac{s^4+6s^2+4}{4s(s^2+1)}V_2 \qquad \ldots(\text{v})$$

Comparing Eqs (iii) and (v) with *Y*-parameter equation,

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{s^3+2s}{4(s^2+1)} & -\dfrac{s^3}{4(s^2+1)} \\ -\dfrac{s^3}{4(s^2+1)} & \dfrac{s^4+6s^2+4}{4s(s^2+1)} \end{bmatrix}$$

23) Determine the transmission parameters for the network shown in Fig



SOL:

Applying KCL at Node 1,

$$I_1 = \frac{V_1}{s} + (V_1 - V_2)$$

$$= \frac{s+1}{s}V_1 - V_2 \qquad \ldots(\text{i})$$

Applying KCL at Node 2,

$$I_2 = \frac{V_2}{\frac{1}{s}} + (V_2 - V_1)$$

$$= (s+1)V_2 - V_1$$

$$V_1 = (s+1)V_2 - I_2 \qquad \ldots(\text{ii})$$

Substituting Eq. (ii) in Eq. (i),

$$I_1 = \frac{s+1}{s}[(s+1)V_2 - I_2] - V_2$$

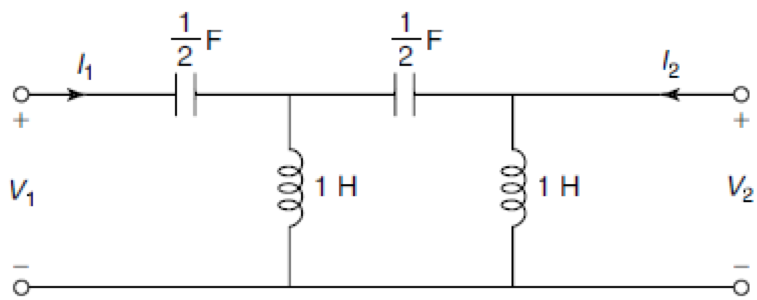
$$= \left[\frac{(s+1)^2}{s} - 1\right]V_2 - \frac{s+1}{s}I_2$$

$$= \left(\frac{s^2+s+1}{s}\right)V_2 - \left(\frac{s+1}{s}\right)I_2 \qquad \ldots(\text{iii})$$

Comparing Eqs (ii) and (iii) with *ABCD* parameter equations,

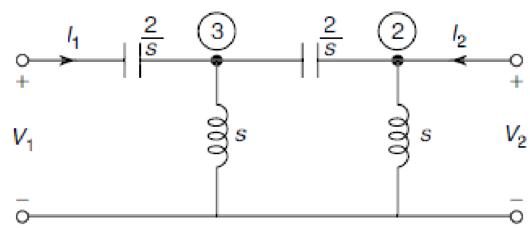
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} s+1 & -1 \\ \frac{s^2+s+1}{s} & \frac{s+1}{s} \end{bmatrix}$$

24) Find h-parameters for the network shown in Fig.



SOL:

The transformed network as shown in Fig.



From Fig.

$$I_1 = \frac{V_1 - V_3}{\frac{2}{s}}$$

$$= \frac{s}{2}V_1 - \frac{s}{2}V_3 \qquad \ldots(\text{i})$$

Applying KCL at Node 3,

$$\frac{s}{2}(V_1 - V_3) = \frac{V_3}{s} + \frac{s}{2}(V_3 - V_2)$$

$$\frac{s}{2}V_3 + \frac{1}{s}V_3 + \frac{s}{2}V_3 = \frac{s}{2}V_1 + \frac{s}{2}V_2$$

$$V_3 = \frac{s^2}{2(s^2+1)}V_1 + \frac{s^2}{2(s^2+1)}V_2 \qquad \ldots(\text{ii})$$

Substituting Eq. (ii) in Eq. (i),

$$I_1 = \frac{s}{2}V_1 - \frac{s}{2}\left[\frac{s^2}{2(s^2+1)}V_1 + \frac{s^2}{2(s^2+1)}V_2\right]$$

$$= \left[\frac{s}{2} - \frac{s^3}{4(s^2+1)}\right]V_1 - \frac{s^3}{4(s^2+1)}V_2$$

$$= \frac{s^3+2s}{4(s^2+1)}V_1 - \frac{s^3}{4(s^2+1)}V_2 \qquad \ldots\text{(iii)}$$

Applying KCL at Node 2,

$$I_2 = \frac{V_2}{s} + \frac{s}{2}(V_2 - V_3)$$

$$= \frac{s^2+2}{2s}V_2 - \frac{s}{2}V_3 \qquad \ldots\text{(iv)}$$

Substituting Eq. (ii) in Eq. (iv),

$$I_2 = \frac{s^2+2}{2s}V_2 - \frac{s}{2}\left[\frac{s^2}{2(s^2+1)}V_1 + \frac{s^2}{2(s^2+1)}V_2\right]$$

$$= -\frac{s^3}{4(s^2+1)}V_1 + \left[\frac{s^2+2}{2s} - \frac{s^3}{4(s^2+1)}\right]V_2$$

$$= -\frac{s^3}{4(s^2+1)}V_1 + \frac{s^4+6s^2+4}{4s(s^2+1)}V_2 \qquad \ldots\text{(v)}$$

$$V_1 = \frac{4(s^2+1)}{s(s^2+2)}I_1 + \frac{s^2}{s^2+2}V_2$$

$$I_2 = -\frac{s^3}{4(s^2+1)}\left[\frac{4(s^2+1)}{s(s^2+2)}I_1 + \frac{s^2}{s^2+2}V_2\right] + \frac{s^4+6s^2+4}{4s(s^2+1)}V_2$$

$$= -\frac{s^2}{s^2+2}I_1 + \frac{2(s^2+1)}{s(s^2+2)}V_2$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{4(s^2+1)}{s(s^2+2)} & \dfrac{s^2}{s^2+2} \\ -\dfrac{s^2}{s^2+2} & \dfrac{2(s^2+1)}{s(s^2+2)} \end{bmatrix}$$

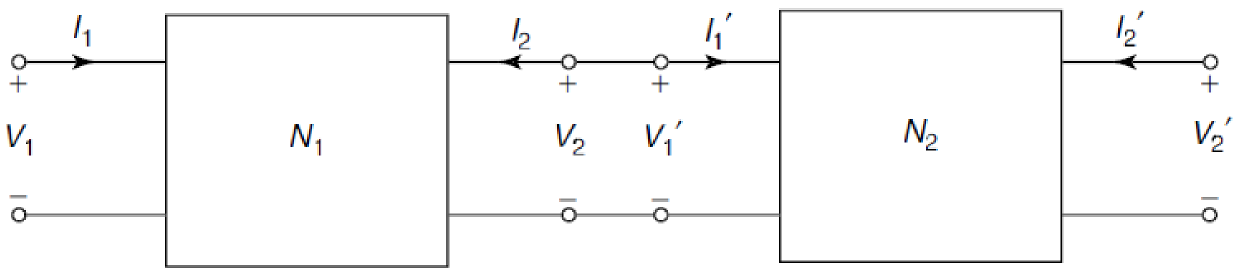
INTERCONNECTION OF TWO-PORT NETWORKS

When the two networks are interconnected, the input and output quantities are to be determined with Cascade Connection, series and parallel connections. The cascade connection is useful to determine ABCD parameters of two interconnected networks; the series connection is useful to determine Z parameters of two interconnected networks; the parallel connection is useful to determine Y parameters of two interconnected networks. With the help of relationship between two-port parameters, the other parameters also are to be determined.

i) CASCADE CONNECTION:

The Figure shows two-port networks connected in cascade. In the cascade connection, the output port of the first network becomes the input port of the second network. Since it is assumed that input and output currents are positive when they enter the network, we have

$$I_1' = -I_2$$



Let A_1, B_1, C_1, D_1 be the transmission parameters of the network N_1 and A_2, B_2, C_2, D_2 be the transmission parameters of the network N_2.

For the network N_1,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

For the network N_2,

$$\begin{bmatrix} V_1' \\ I_1' \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix}$$

Since $V_1' = V_2$ and $I_1' = -I_2$, we can write

$$\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix}$$

Combining Eqs

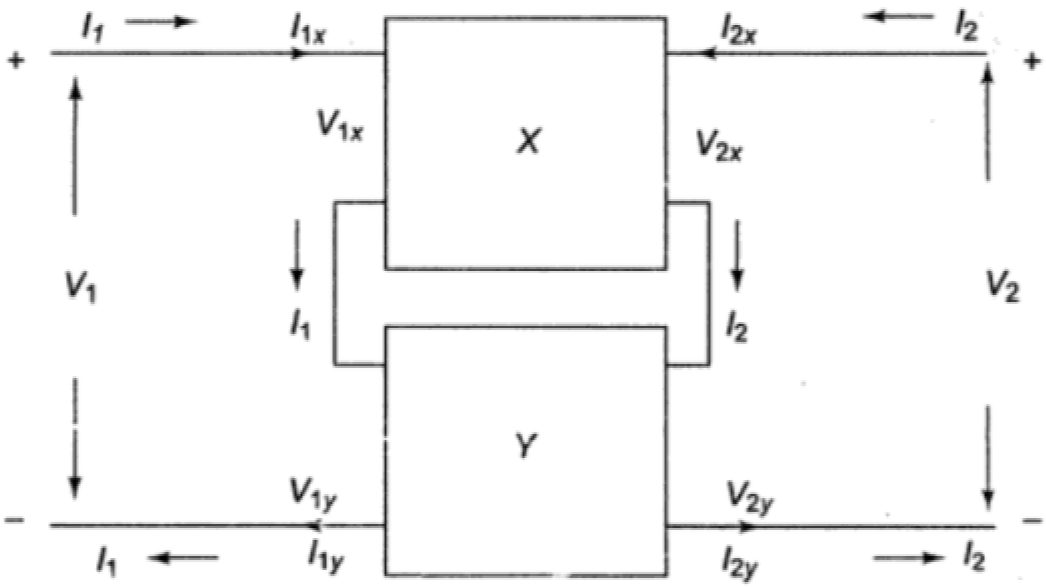
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix}$$

Hence, $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$

The above equation shows that the resultant ABCD matrix of the cascade connection is the product of the individual ABCD matrices.

ii) SERIES CONNECTION:

The Figure shows two-port networks connected in series. In a series connection, both the networks carry the same input current. Their output currents are also equal.



Let us consider two, two-port networks, connected in series as shown. If each port has a common reference node for its input and output, and if these references are connected together then the equations of the networks X and Y in terms of Z parameters are

$$V_{1X} = Z_{11X} I_{1X} + Z_{12X} I_{2X}$$

$$V_{2X} = Z_{21X} I_{1X} + Z_{22X} I_{2X}$$

$$V_{1Y} = Z_{11Y} I_{1Y} + Z_{12Y} I_{2Y}$$

$$V_{2Y} = Z_{21Y} I_{1Y} + Z_{22Y} I_{2Y}$$

From the inter-connection of the networks, it is clear that

$$I_1 = I_{1X} = I_{1Y}; I_2 = I_{2X} = I_{2Y}$$

and $V_1 = V_{1X} + V_{1Y}; V_2 = V_{2X} + V_{2Y}$

$$\therefore \quad V_1 = Z_{11X} I_1 + Z_{12X} I_2 + Z_{11Y} I_1 + Z_{12Y} I_2$$

$$= (Z_{11X} + Z_{11Y}) I_1 + (Z_{12X} + Z_{12Y}) I_2$$

$$V_2 = Z_{21X} I_1 + Z_{22X} I_2 + Z_{21Y} I_1 + Z_{22Y} I_2$$

$$= (Z_{21X} + X_{21Y}) I_1 + (Z_{22X} + Z_{22Y}) I_2$$

The describing equations for the series connected two-port network are

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

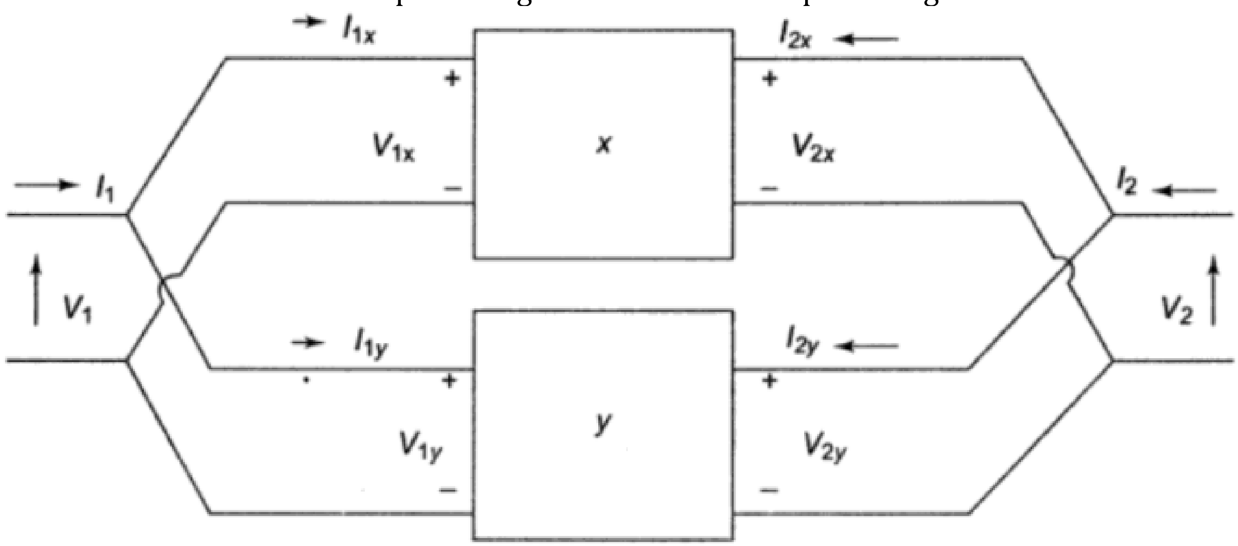
where $Z_{11} = Z_{11X} + Z_{11Y}$; $\quad Z_{12} = Z_{12X} + Z_{12Y}$

$Z_{21} = Z_{21X} + Z_{21Y}$; $\quad Z_{22} = Z_{22X} + Z_{22Y}$

Thus, the resultant Z-parameter matrix for the series-connected networks is the sum of Z-parameters of each individual two-port network.

iii) PARALLEL CONNECTION:

The Figure shows two-port networks connected in parallel. In the parallel connection, the two networks have the same input voltages and the same output voltages.



$$I_{1X} = Y_{11X} V_{1X} + Y_{12X} V_{2X}$$

$$I_{2X} = Y_{21X} V_{1X} + Y_{22X} V_{2X}$$

$$I_{1Y} = Y_{11Y} V_{1Y} + Y_{12Y} V_{2Y}$$

$$I_{2Y} = Y_{21Y} V_{1Y} + Y_{22Y} V_{2Y}$$

From the interconnection of the networks, it is clear that

$$V_1 = V_{1X} = V_{1Y}; V_2 = V_{2X} = V_{2Y}$$

and $I_1 = I_{1X} + I_{1Y}; I_2 = I_{2X} + I_{2Y}$

$$\therefore \quad I_1 = Y_{11X} V_1 + Y_{12X} V_2 + Y_{11Y} V_1 + Y_{12Y} V_2$$

$$= (Y_{11X} + Y_{11Y}) V_1 + (Y_{12X} + Y_{12Y}) V_2$$

$$I_2 = Y_{21X} V_1 + Y_{22X} V_2 + Y_{21Y} V_1 + Y_{22Y} V_2$$

$$= (Y_{21X} + Y_{21Y}) V_1 + (Y_{22X} + Y_{22Y}) V_2$$

The describing equations for the parallel connected two-port networks are

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

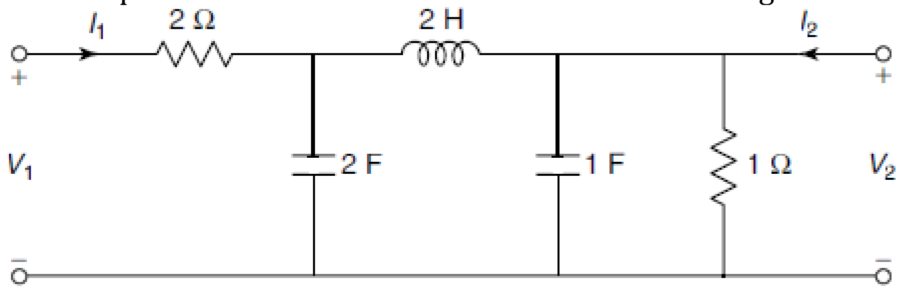
where $Y_{11} = Y_{11X} + Y_{11Y}; \qquad Y_{12} = Y_{12X} + Y_{12Y}$

$$Y_{21} = Y_{21X} + Y_{21Y}; \qquad Y_{22} = Y_{22X} + Y_{22Y}$$

Thus we see that each Y parameter of the parallel network is given as the sum of the corresponding parameters of the individual networks.

PROBLEMS ON INTERCONNECTED NETWORKS

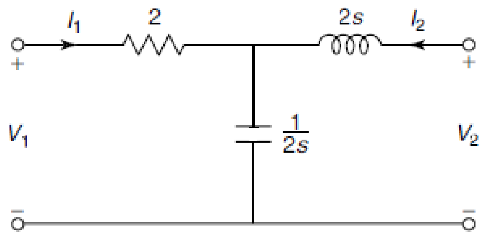
25) Determine ABCD parameters for the ladder network shown in Fig.



SOL:

The above network can be considered as a cascade connection of two networks N_1 and N_2.

The network N_1 is shown in Fig.



Applying KVL to Mesh 1,

$$V_1 = \left(2 + \frac{1}{2s}\right) I_1 + \frac{1}{2s} I_2 \qquad \ldots(i)$$

Applying KVL to Mesh 2,

$$V_2 = \frac{1}{2s} I_1 + \left(2s + \frac{1}{2s}\right) I_2 \qquad \ldots(ii)$$

From Eq. (ii),

$$I_1 = 2s\, V_2 - (4s^2 + 1)\, I_2 \qquad \ldots(iii)$$

Substituting Eq. (iii) in Eq. (i),

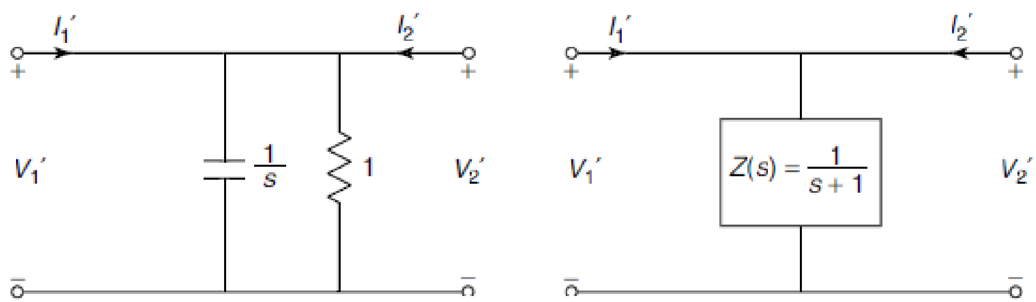
$$V_1 = \left(2 + \frac{1}{2s}\right)[2s\, V_2 - (4s^2 + 1) I_2] + \frac{1}{2s} I_2$$

$$= (4s + 1) V_2 - (8s^2 + 2s + 2) I_2 \qquad \ldots(iv)$$

Comparing Eqs (iv) and (iii) with *ABCD* parameter equations,

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} 4s+1 & 8s^2 + 2s + 2 \\ 2s & 4s^2 + 1 \end{bmatrix}$$

The network N_2 is shown in Fig.



(a) (b)

Applying KVL to Mesh 1,

$$V_1' = \frac{1}{s+1} I_1' + \frac{1}{s+1} I_2' \qquad \ldots(i)$$

Applying KVL to Mesh 2,

$$V_2' = \frac{1}{s+1} I_1' + \frac{1}{s+1} I_2' \qquad \ldots(ii)$$

From Eq. (ii),

$$I_1' = (s+1)V_2' - I_2' \qquad \text{...(iii)}$$

Also, $$V_1' = V_2' \qquad \text{...(iv)}$$

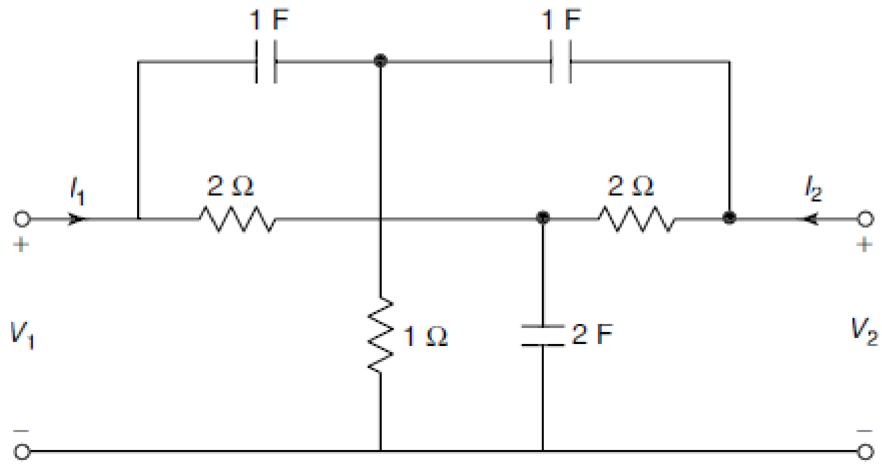
Comparing Eqs (iv) and (iii) with *ABCD* parameter equations,

$$\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ s+1 & 1 \end{bmatrix}$$

Hence, overall *ABCD* parameters are

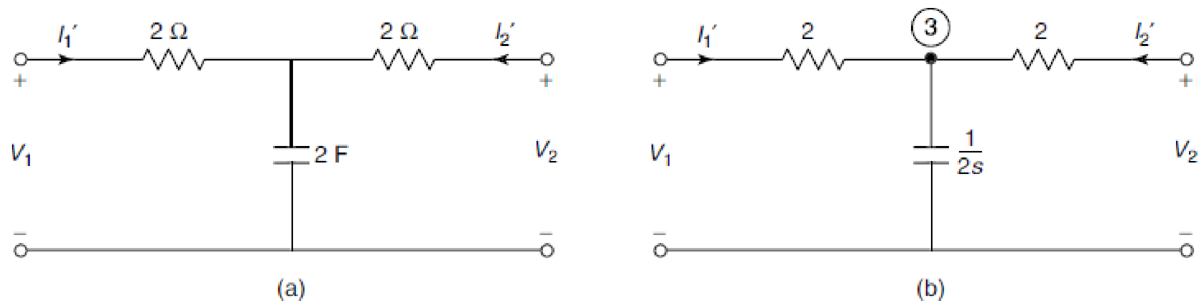
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 4s+1 & 8s^2+2s+2 \\ 2s & 4s^2+1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ s+1 & 1 \end{bmatrix} = \begin{bmatrix} 8s^3+10s^2+8s+3 & 8s^2+2s+2 \\ 4s^3+4s^2+3s+1 & 4s^2+1 \end{bmatrix}$$

26) Find Y-parameters for the network shown in Fig.



SOL:

The above network can be considered as a parallel connection of two networks, N_1 and N_2. The network N_1 is shown in Fig.



Applying KCL at Node 3,

$$I_1' + I_2' = 2s\,V_3 \qquad \text{...(i)}$$

From Fig.

$$I_1' = \frac{V_1 - V_3}{2}$$

$$= \frac{1}{2}V_1 - \frac{1}{2}V_3 \qquad \text{...(ii)}$$

$$I_2' = \frac{V_2 - V_3}{2}$$

$$= \frac{1}{2}V_2 - \frac{1}{2}V_3 \qquad \text{...(iii)}$$

Substituting Eq. (ii) and Eq. (iii) in Eq. (i),

$$\frac{V_1}{2} - \frac{V_3}{2} + \frac{V_2}{2} - \frac{V_3}{2} = (2s)\,V_3$$

$$(2s+1)V_3 = \frac{V_1}{2} + \frac{V_2}{2}$$

$$V_3 = \frac{1}{2(2s+1)}V_1 + \frac{1}{2(2s+1)}V_2 \qquad \text{...(iv)}$$

Substituting Eq. (iv) in Eq. (ii),

$$I_1' = \frac{V_1}{2} - \frac{1}{2}\left[\frac{1}{2(2s+1)}V_1 + \frac{1}{2(2s+1)}V_2\right]$$

$$= \left(\frac{4s+1}{8s+4}\right)V_1 - \left(\frac{1}{8s+4}\right)V_2 \qquad \text{...(v)}$$

Substituting Eq. (iv) in Eq. (iii),

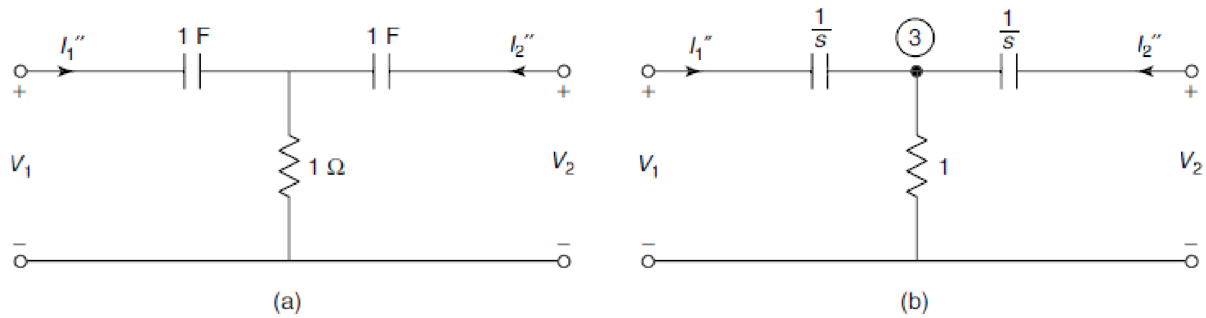
$$I_2' = \frac{V_2}{2} - \frac{1}{2}\left[\frac{1}{2(2s+1)}V_1 - \frac{1}{2(2s+1)}V_2\right]$$

$$= -\left(\frac{1}{8s+4}\right)V_1 + \left(\frac{4s+1}{8s+4}\right)V_2 \qquad \text{...(vi)}$$

Comparing Eqs (v) and (vi) with Y-parameter equations,

$$\begin{bmatrix} Y_{11}' & Y_{12}' \\ Y_{21}' & Y_{22}' \end{bmatrix} = \begin{bmatrix} \dfrac{4s+1}{8s+4} & -\dfrac{1}{8s+4} \\ -\dfrac{1}{8s+4} & \dfrac{4s+1}{8s+4} \end{bmatrix}$$

The network N_2 is shown in Fig.



Applying KCL at Node 3,

$$I_1'' + I_2'' = V_3 \qquad \ldots(i)$$

From Fig.

$$I_1'' = \frac{V_1 - V_3}{\frac{1}{s}}$$

$$= sV_1 - sV_3 \qquad \ldots(ii)$$

$$I_2' = \frac{V_2 - V_3}{\frac{1}{s}}$$

$$= sV_2 - sV_3 \qquad \ldots(iii)$$

Substituting Eqs (ii) and (iii) in Eq. (i),

$$sV_1 - sV_3 + sV_2 - sV_3 = V_3$$

$$(2s+1)V_3 = sV_1 + sV_2$$

$$V_3 = \left(\frac{s}{2s+1}\right)V_1 + \left(\frac{s}{2s+1}\right)V_2 \qquad \ldots(iv)$$

Substituting Eq. (iv) in Eq. (ii),

$$I_1'' = sV_1 - s\left[\left(\frac{s}{2s+1}\right)V_1 + \frac{s}{(2s+1)}V_2\right]$$

$$= \left[\frac{s(s+1)}{2s+1}\right]V_1 - \left(\frac{s^2}{2s+1}\right)V_2 \qquad \ldots(v)$$

Substituting Eq. (iv) in Eq. (iii),

$$I_2'' = sV_2 - s\left[\left(\frac{s}{2s+1}\right)V_1 + \left(\frac{s}{2s+1}\right)V_2\right]$$

$$= -\left(\frac{s^2}{2s+1}\right)V_1 + \left[\frac{s(s+1)}{2s+1}\right]V_2 \qquad \ldots(vi)$$

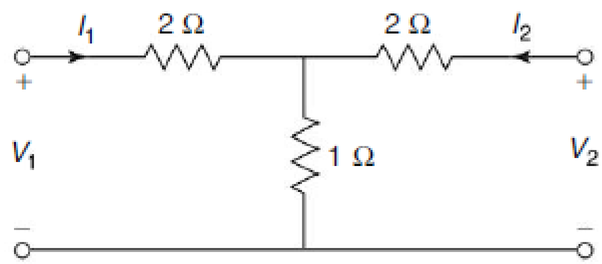
Comparing Eqs (v) and (vi) with Y-parameter equations,

$$\begin{bmatrix} Y_{11}'' & Y_{12}'' \\ Y_{21}'' & Y_{22}'' \end{bmatrix} = \begin{bmatrix} \dfrac{s(s+1)}{2s+1} & -\left(\dfrac{s^2}{2s+1}\right) \\ -\left(\dfrac{s^2}{2s+1}\right) & \dfrac{s(s+1)}{2s+1} \end{bmatrix}$$

Hence, the overall Y-parameters of the network are

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21}' & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_{11}' + Y_{11}'' & Y_{12}' + Y_{12}'' \\ Y_{21}' + Y_{21}'' & Y_{22}' + Y_{22}'' \end{bmatrix} = \begin{bmatrix} \dfrac{4s^2+8s+1}{4(2s+1)} & -\dfrac{(4s^2+1)}{4(2s+1)} \\ -\dfrac{(4s^2+1)}{4(2s+1)} & \dfrac{4s^2+8s+1}{4(2s+1)} \end{bmatrix}$$

27) Two identical sections of the network shown in Fig. are connected in series. Obtain Z-parameters of the overall connection.



SOL:

Applying KVL to Mesh 1,

$$V_1 = 3I_1 + I_2 \qquad \ldots(\text{i})$$

Applying KVL to Mesh 2,

$$V_2 = I_1 + 3I_2 \qquad \ldots(\text{ii})$$

Comparing Eqs (i) and (ii) with Z-parameter equations,

$$\begin{bmatrix} Z_{11}'' & Z_{12}'' \\ Z_{21}'' & Z_{22}'' \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

Hence, Z-parameters of the overall connection are

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$$